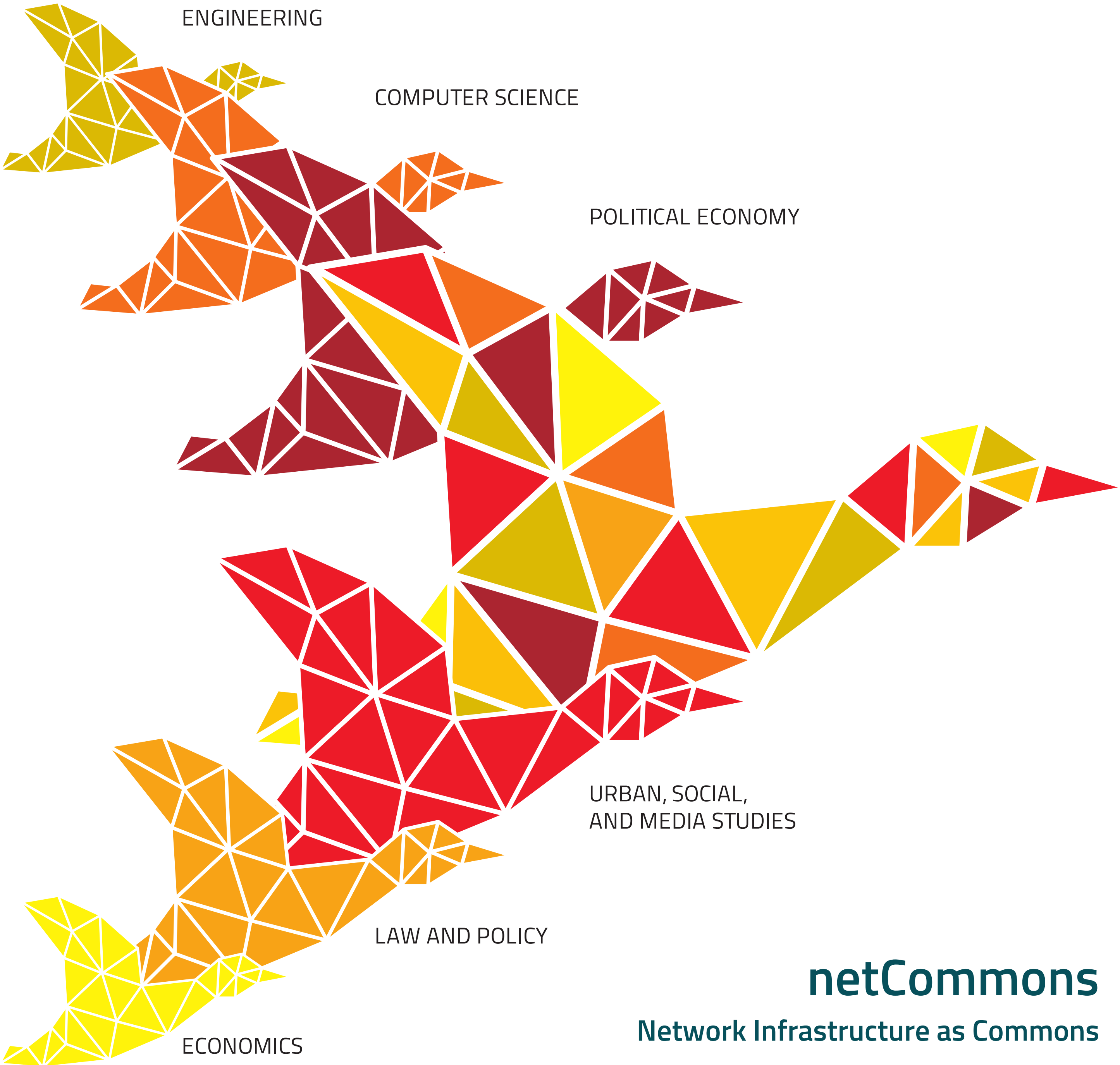

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Network Infrastructure as Commons

Incentives for Participation and Active Collaboration in CNs

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Executive summary

D2.8 is the follow-up deliverable of D2.3, entitled "Incentives for Participation and Active Collaboration in CNs" [1]. In [1], the goal has been to provide a thorough review of what serves as participation and collaboration incentive in CNs. Throughout that work, our interpretation of the term "incentive" is quite relaxed and inclusive, embracing political causes, cultural aspects and social effects.

In D2.8, the departure from its predecessor deliverable is two-fold. First, the emphasis is on incentive mechanisms the way the term is used by the economics community. Hence, we propose, present, and analyze ways to share the deployment and operational costs of a CN as well as the possible revenues from it (*e.g.*, pricing policies, subscription schemes, investment cost sharing rules) that incentivize the engagement of different CN stakeholders.

Second, the carried out and reported research work is inspired by concrete CN instances. These serve as case studies and are highly representative of the broad variety CNs exhibit with respect to the different types of actors involved in them and the environment in which they grow (rural *vs.* urban). Hence, our research work is inspired by the economics (one could use the term "business models") of three CN instances: the B4RN CN in Lancashire, UK, the guifi.net CN in Catalonia, Spain, and the Sarantaporo.gr CN, in Greece. The first two CNs represent CN success stories in Europe and worldwide, with strong elements of novelty in the chosen technology, business model, and strategy. The third CN is a younger initiative, which has been strongly dependent on public subsidy during its deployment phase, and has now been experimenting with novel subscription schemes to render its operation self-sustainable.

Having said that, the relevance of the reported work is by no means limited to the specific CNs. On the contrary, both the models and the proposed/analyzed mechanisms can find application to many more CNs across the world, possibly with adaptations accounting for the particularities and constraints of the specific environment. In fact, a more ambitious objective of this work is exactly to develop elements of a theory for the economics of community networks. Such a theory has been missing so far since in very few cases the deployment of these CNs has followed some "business plan". As many of these initiatives mature and questions of scalability and sustainability become more relevant, people running these networks will increasingly pursue insights and guidelines for sustainably funding the deployment and operation of their CNs. This deliverable could be seen as one of the first attempts to: (a) identify existing relevant research work in economics, primarily, and informatics; and (b) carry out novel research, that could serve this purpose.

The deliverable is structured into four main chapters. In chapter 2, we summarize a model for determining optimal subscription prices for wireless CNs [2]. The interest in the model is two-fold: first, it captures the evolutionary growth of wireless community networks, giving insights to the relation between the original investment on the network infrastructure and proper pricing policies. Secondly, we reuse it as a component of our modeling work in chapter 5. In chapter 3, we focus on the distinct model of the B4RN CN for deploying fiber cable infrastructure and propose the truthful cost sharing mechanism in [3] as a tool for managing the funding of the project and concluding upon its economic sustainability. Then, chapters 4 and 5 present fully original work. In chapter 4, we present an innovative subscription scheme that has been introduced in the Sarantaporo.gr CN as a means to motivate the engagement of the community in its funding. We summarize main hints about its operation, while its full analysis is provided in a technical report (submitted and currently under review) that is added as an appendix to the deliverable. Finally, in chapter 5, we focus on guifi.net and its innovative approaches towards establishing synergies with for-profit entities. We propose and analyze models for such synergies in two different scenarios: (a) the for-profit actors provide Internet access over the shared community network infrastructure; and (b) the shared resources are storage space and computational capacity. In particular, we explore the suitability of different cost-sharing mechanisms regarding the equilibrium strategies they induce in the games that emerge for the involved actors.

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List of Acronyms

ACP	Average Cost Pricing
CAPEX	Capital Expenses
CN	Community Network
CNO	Community Network Operator
CPR	Common Pool Resources
EU	European Union
MNO	Mobile Network Operator
MVNO	Mobile Virtual Network Operator
NE	Nash Equilibrium
NP	Network Provider
OPEX	Operational Expenses
PIP	Physical Infrastructure Provider
SCS	Serial Cost Sharing
SP	Service Provider

1. Introduction

1.1. From the big picture of D2.3 to the case studies of D2.8

D2.8 is the second deliverable under Task 2.2 of the project, following D2.3 [1]. In D2.3, we start from the different organizational structures witnessed across CN instances and identify the different types of stakeholders that are relevant to them (users, volunteers, commercial service providers *a.k.a* professionals, public authorities). We then classify the set of incentives into different categories (political, socio-cultural, and economic), assessing their relevance to each distinct stakeholder type. Throughout D2.3 work, our interpretation of the term "incentive" is quite relaxed and inclusive, embracing political causes, cultural aspects and social effects.

Moreover, we explicitly distinguish between "incentives", *i.e.*, the implicit motives that urge people's involvement in a CN (*e.g.*, their desire for autonomy or the do-it-yourself culture) and "incentive mechanisms" that are put in place by CNs and the teams managing them to respond to these motives (distributed open decision-making processes or do-it-yourself kits for setting up nodes and antennas, respectively). Again, the context attributed to the term "incentive mechanism" is broad, in line with our relaxed interpretation of "incentives". Besides mechanisms already implemented and tested in operational CNs, we also discuss in D2.3 promising mechanisms proposed in the scientific literature. An indicative taxonomy of the full set of incentive mechanisms (both proposed-only and implemented) per stakeholder type they address is provided in Table 1.1.

In D2.8, the departure from its predecessor deliverable D2.3 is two-fold. First, the emphasis is on incentive mechanisms, the way the term is used by the economics community. Hence, we propose, present, and analyze mechanisms to share the deployment or operational costs of a CN as well as the revenues from their operation (*e.g.*, pricing policies, subscription schemes, investment cost sharing rules) that incentivize the engagement of different CN stakeholders.

Secondly, our research work is more case-study driven compared to D2.3. One thing made clear during the review work on incentives and incentive mechanisms in D2.3 is that CNs differentiate strongly, not least due to the different types of involved stakeholders in each case and the environment in which they grow (rural *vs.* urban). Hence, our research work is inspired by the economics, one could call them "business models", of three CN instances: the B4RN CN in Lancashire, UK, the guifi.net in Catalonia, Spain, and the Sarantaporo.gr CN, in Greece. The first two CNs represent CN success stories in Europe and worldwide, with strong elements of novelty in the used technology, business model, and strategy. The third CN is a recently launched activity which has been strongly dependent on public subsidy in its deployment phase, and has now been experimenting with novel subscription schemes to render its operation sustainable.

That said, the reported work is by no means strictly relevant to the specific CNs. On the contrary, both the models and the proposed/analyzed mechanisms can find application to the broader bulk of CNs across the world, possibly with adaptations accounting for the particularities and constraints of the specific environment. In fact, a more ambitious objective of this work is to develop elements of a theory for the economics of community networks. Such a theory has been missing so far since in very few cases the deployment of these CNs has followed some "business plan". As many of these initiatives mature and questions of scalability and sustainability become more relevant, people running these networks will increasingly pursue insights and guidelines for sustainably funding the deployment and operation of their CNs. This deliverable could be seen as one of the first attempts to identify existing work in economics, primarily, and informatics and develop new one that could serve this purpose.

Mechanisms	Volunteers	Users	Commercial service providers	Public administrations
Direct reciprocity		x		
Indirect reciprocity		x		
Punishment of free-riders	x			
Community currencies		x	x	
Donation certificates			x	x
Financial compensation			x	
Local data storage infrastructure		x		
Social events and meetings	x	x		
New member induction processes		x		
Workshops and seminars		x		
Online material for DIY fans		x		
Local applications and services		x		
Operation as legal entities		x	x	x
Licenses and Agreements		x	x	

Table 1.1: Incentives mechanisms and relevance to stakeholders.

1.2. Background

As with standard networks and more general businesses, we can look into the expenses and revenues of CNs. The former can be separated into capital expenditure and operating expenses.

1.2.1. Capital expenditure *vs.* operational expenses

What constitutes Capital Expenditure (CAPEX) for CNs is closely related to the technology used for launching the CN. Most CNs depend on wireless networking technologies (almost always WiFi, much more rarely cellular technologies), hence CAPEX involves the cost of wireless access points/routers and antennas that make up the wireless CN, edge routers that route traffic from/towards the CN, servers that are used for user authentication, data storage and other service provision purposes. Installation costs also come under CAPEX and include the cost of mounting antennas and WiFi equipment. In many CNs, this cost is undertaken by volunteers who assist new members when they join the CN.

When CNs are fully built on fiber, as the case is with several CNs in US and B4RN in UK (ref. section 3), the CAPEX include the digging costs and rights-of-way for laying out fiber-optic cables. In general, these can be orders of magnitude higher than in a wireless CN, but CNs have demonstrated their potential to leverage the social dimension of the addressed communities to decrease it.

Operational expenses (OPEX), on the other hand, relate to the cost of peering agreements for Internet access (cost of leased lines), the maintenance of network nodes, the use of software for network management, monitoring and billing purposes, the salaries of (permanent) personnel, as well as electricity costs. Compared to standard for-profit networks, CNs can exhibit considerably reduced expenditure, either due to the use of wireless technology, where this is the case, or mobilizing resources from volunteers.

1.2.2. Evolutionary *vs.* up-front deployment of network infrastructure

The CN infrastructure may be deployed in two ways.

More often than not, this happens in an *evolutionary* manner. First, an initial investment helps setting up a small network, namely a few nodes connected with each other together with a server that lets manage them.

This investment is typically made by the group of volunteers that launch the CN. In several cases, these initial resources are the result of some grant, either monetary or in-kind, most often by public or private non-profit institutes in the context of social cohesion projects. Then, over time, community members joining the CN add their own nodes to it, expanding its coverage and bringing it closer to more users, who were earlier out of its range. This evolutionary process is typical of the wireless community networks, which form the majority of CNs in Europe and worldwide.

More rarely, the whole network infrastructure is built *up front*, after securing the funding of the infrastructure deployment project by the community. This is the case with B4RN, which digs and lays out fiber-optic cables across the whole village, if the local community decides during a preceding consultation process to move forward with the project.

1.2.3. Revenue sources

Finally, the possible revenue sources for CNs are detailed in [5]. We summarize them below for the sake of self-completeness:

- Member subscriptions and contributions in kind, as in network equipment, time, effort. Member subscriptions may be mandatory or voluntary. This source has proven by far the more sustainable and reliable up to now and supports most of the successful CNs in Europe, which count tens of thousands of nodes.
- Donations from supporters in the context of crowd-funding campaigns, regular or one-time donations.
- Support from public agencies (*e.g.*, municipalities, European Union) and non-profit institutions through social cohesion or research projects.
- Funding from private sector through commons-based policies that foster synergies with entities undertaking commercial for-profit activities [6].

1.3. Deliverable outline

In chapter 2, we summarize a model for determining optimal subscription prices for CNs that grows in evolutionary manner. The model is presented in [2] and, to the best of our knowledge, is the first one that captures the evolutionary growth of wireless community networks, giving insights to the relation between the original investment on the network infrastructure and the proper pricing policies. In chapter 3, we focus on the distinct model of B4RN for up-front deploying CN infrastructure and point to the truthful cost sharing mechanism in [3] for guiding the two main decisions that have to be made before launching the infrastructure deployment: whether such a project is worth carrying out and how should its costs be split among the community members.

Then, chapters 4 and 5 present fully original work. Chapter 4 is devoted to an innovative subscription scheme that has been introduced in the Sarantaporo.gr for motivating the participation of the community in its funding. We present the scheme and summarize main guidelines for tuning it, while we defer its full analysis for a full report provided as appendix to the deliverable. Finally, in chapter 5, we focus on guifi.net and its innovative approaches towards establishing synergies with for-profit entities. We propose and analyze models for such synergies when (a) the for-profit actors provide Internet access over the shared community network infrastructure; and (b) the shared infrastructure is storage space and computational capacity. In particular, we explore the suitability of different cost-sharing mechanisms regarding the equilibrium strategies they induce in the games that emerge for the involved actors

1.4. Impact - contributions

Summarizing, the intended contributions of the work reported in this deliverable are:

- Practical hints and guidelines towards specific CNs as to how to optimize their funding process. This is the case with the collective subscriptions scheme in the Sarantaporo.gr or the cost-sharing mechanism in operation in the guifi.net CN
- The support with technical data of the arguments that CNs formulate (or should formulate) in favor of their sustainability and long-term potential. Such arguments typically target regulatory and policy-making bodies.
- Elements of what could evolve to a theory of crowdsourced network infrastructures. Such a theory is expected to become more relevant in view of the huge investments implied by current and future agendas for broadband and mobile connectivity worldwide.

2. Pricing policies for sustainable wireless community networks

2.1. Motivation-inspiration

The majority of community initiatives are wireless community networks since the use of wireless technologies (almost always WiFi) reduces considerably the cost required for setting and operating a community network. The first reason for this cost reduction relates to the possibility to use the unlicensed frequency bands that are allocated worldwide to WiFi systems. The second reason relates to the significantly smaller cost of network equipment (when compared to the layout of fiber, see chapter 3). and the way the network can grow in an evolutionary way. Typically, an initial investment covers the cost of deploying the first few nodes that provide coverage to a small geographical area compared to the geographical spread of the community. This investment is undertaken by a small team of volunteers who lead the CN initiative or with funds from a dedicated project, *e.g.*, a social cohesion project or research grant. Then, community members add their own, generally low-cost equipment (an antenna and an access point), thus undertaking their small share of the network infrastructure cost as they deem it worthwhile.

Under the favorable scenario, the CN progressively registers more members and the CN grows to an operationally *stable* point that provides coverage to the whole community, or a large part of it. On the other hand, there are plenty of CN stories that followed different trajectories towards extinction. This may be because they never captured the interest of the local community, or because they captured it only temporarily till competing solutions for network connectivity became available from commercial operators in their area. Relevant empirical findings are documented in [7], which identifies typical patterns and anti-patterns in the life of a CN, some of which refer to the Economic compensation or the Dumping problem (see sect. 4 in [7]).

The work in [2] tried to capture the evolutionary way CNs grow and the multiple trajectories they might follow towards success or failure. It is, to the best of our understanding, the first and main work that addresses the evolutionary dynamics of the wireless CNs. At the same time, it both complements and contrasts with original work we have carried out in subsequent chapters. Hence, in what follows, we briefly present the analysis and its findings about pricing strategies and their relation to the initial investment in wireless CNs.

2.2. System model

The different actor entities in the system model are given in Fig. 2.1 and discussed in the next sections.

2.2.1. CN operator

The term denotes the small group of people who have initiated and typically operate the CN. They are often organized as a non-profit entity and one of their main tasks is to ensure the sustainable funding of the effort. The CNO carries out the initial investment in the network, setting up the first network nodes. It then undertakes maintenance and user/node management operations and is also responsible for buying Internet (IP) transit from commercial upstream ISPs at a (monthly) cost c_s.

In return, it receives a subscription fee, f_s, from each CN user, which is typically much smaller than what a commercial operator would charge. These fees make up for the operational expenses of the network and fund

Figure 2.1: The typical stakeholders in a wireless community network.

its further growth. Hence, the total revenue of the CNO from the CN operations is

$$R_{CNO} = N \cdot n_{sub} \cdot f_s - c_s$$

where n_{sub} is the portion of subscribers to the CN out of N, the pool of users (community members). Both the number of CN subscribers and the subsequent revenue of the CNO are functions of time, *i.e.*, $n_{sub}(t)$ and $R_{CNO}(t)$.

2.2.2. CN users

Let $\mathcal{N}$ be the set of community members (*e.g.*, in households), with $N = |\mathcal{N}|$. This serves essentially as an upper bound on the number of subscriptions the CN can register.

Users decide whether to join the CN taking into account different criteria. In [2], two such criteria are considered, the network coverage Cov(t) and the subscription fee f_s. When a user joins the CN, she obtains a utility (payoff) that equals

$$poff_u = a_u \cdot Cov - f_s \tag{2.1}$$

Users differentiate with each other with respect to the parameter $0 \leq a_u \leq 1$, reflecting how much a user weighs the network coverage. A higher a_u value implies that a given network coverage is appreciated more and that u is willing to pay more for what the network offers. In [2], two assumptions are made for the sake of model tractability:

- The parameter a_u varies uniformly across users between a lower bound α and an upper bound β, $a_u \sim \mathcal{U}(\alpha, \beta)$. Two cases are distinguished with respect to the breadth of the uniform distribution, see Fig. 2.2a.
  - $\beta \leq 2 \cdot \alpha$ and the distribution is called *narrow*.
  - $\beta > 2 \cdot \alpha$ and the distribution is called *wide*.
- The network coverage is a linear function of the number of subscribers, $Cov(t) \propto n_{sub}(t)$, as shown in Fig. 2.2b.

Upon discrete time epochs, t=0, T, 2T,... community members decide whether to join, stay with, or leave the CN.

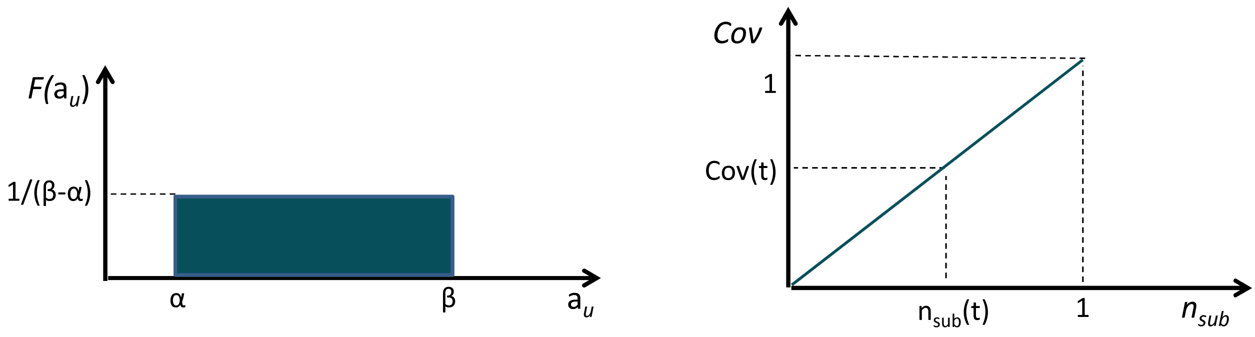


Figure 2.2: Uniform distribution of a_u values (left) and linear dependence of coverage on number of subscribers (right)

.

2.3. Analysis

The CN coverage evolves over epochs t=0, T, 2T,... (we set T=1 to simplify the notation) according to

$$Cov(t) = \frac{1}{\beta - \alpha} \max\{0, \beta - \max\{\alpha, \frac{f_s}{Cov(t-1)}\}\} \tag{2.2}$$

At any point in time, for given so-far coverage $Cov(t-1)$, users with adequately high a_u values are subscribers to the CN and other with low a_u values stay out. At the two extreme points, all users or none subscribe to the CN (see Fig. 2.3), we are interested to understand if there are other equilibrium points.

Note that:

- if at time t-1, $Cov(t-1) = 0$, then $Cov(t) = 0$ and no user will ever have positive

$$poff_u(t) = a_u \cdot Cov(t-1) - f_s \tag{2.3}$$

  in order to join the CN. The CN dies out.
- if at time t-1, $f_s/Cov(t-1) \leq \alpha$, then $Cov(t) = 1$ and all community members subscribe to the CN.
- otherwise, for given f_s, α, β and $Cov(t-1)$, there are one or more equilibrium points Cov_{eq} satisfying

$$(\beta - \alpha) \cdot Cov_{eq}^2 - \beta \cdot Cov_{eq} + f_s = 0 \tag{2.4}$$

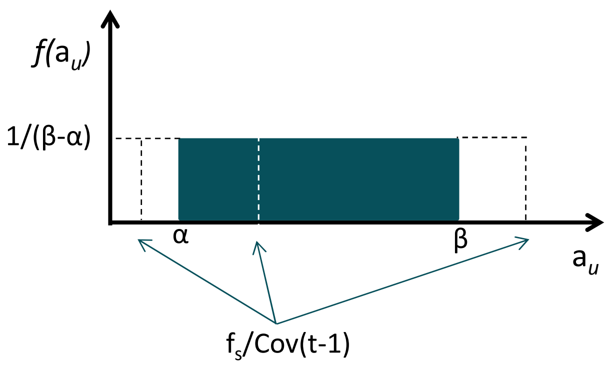


Figure 2.3: At any point in time t, the combination of the (static) subscription fee f_s and the current CN coverage $Cov(t-1)$ determines which users will be subscribers of the CN over the interval [t,t+1].

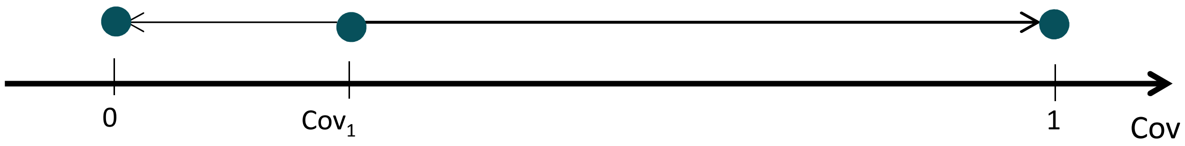


Figure 2.4: Evolution of CN coverage under narrow distribution of CN users and subscription fee $f_s \leq \alpha$.

Now, depending on the solutions of (2.4) and whether the distribution of a_u values is narrow or wide, the analysis identifies the following cases:

Case 1: $0 < f_s \leq \alpha$**, both narrow and wide user distributions.** There are three possible equilibrium points $\{0, Cov_1, 1\}$, with

$$Cov_1 = \frac{\beta - \sqrt{\beta^2 - 4(\beta - \alpha)}}{2(\beta - \alpha)} \tag{2.5}$$

The equilibrium point Cov_1 is *unstable*. Unless $Cov(t-1) = Cov_1$ exactly, so that $Cov(t) = Cov_1$ for all $t \geq t-1$, any small drift of the CN coverage beyond or below Cov_1 results in CN coverage 1 or 0, respectively, as shown in 2.4.

To see this by means of a toy example, consider that $\alpha = 4$, $\beta = 7$, and $f_s = 3$. These values yield $Cov_1 = 0.5657$, from (2.5).

If at time t=0, $Cov(0) = Cov_1 - 0.01 = 0.5557$, the values $Cov(t)$ assumes over subsequent time intervals are

t	0	T	2T	3T	4T
Cov(t)	0.5557	0.5339	0.4604	0.1615	0

Whereas, if at time t=0, the CN coverage is $Cov(0) = Cov_1 + 0.01 = 0.5757$, $Cov(t)$ evolves as follows.

t	0	T	2T	3T	4T
Cov(t)	0.5757	0.5964	0.6567	0.8106	1

Case 2: $f_s > \alpha$**, narrow user distribution.** Both solutions of Eq. (2.4) are > 1. Whichever value $Cov(t-1)$ assumes, over time it converges to zero, which is the single stable equilibrium value.

Case 3: $\alpha < f_s < \frac{\beta^2}{4(\beta-\alpha)}$**, wide user distribution.** In this case, Eq. (2.4) has two solutions, Cov_1, given by (2.5) and Cov_2, given by:

$$Cov_2 = \frac{\beta - \sqrt{\beta^2 - 4(\beta - \alpha)f_s}}{2(\beta - \alpha)} \tag{2.6}$$

There are three equilibrium points, two of which are stable (0, Cov_2) and two are unstable (Cov_1, 1), as Fig. 2.5 demonstrates.

Case 4: $f_s = \frac{\beta^2}{4(\beta-\alpha)}$**, wide user distribution.** This is an extreme case, where the two solutions of (2.4) coincide to a single solution $Cov_{12} = \frac{\beta}{2(\beta-\alpha))}$. There are two equilibrium points, (0, Cov_{12}). If the network coverage ever gets a value $Cov(t) < Cov_{12}$, the CN follows a trajectory of coverage values tending to zero. If it gets any value higher than Cov_{12}, then it stabilizes to Cov_{12} (see Fig. 2.6).

Case 5: $f_s > \frac{\beta^2}{4(\beta-\alpha)}$**, wide user distribution, and** $f_s > \alpha$**), narrow user distribution.** In these cases, the expression within the square root in (2.5) and (2.6) becomes negative. The unique equilibrium lies at point zero.

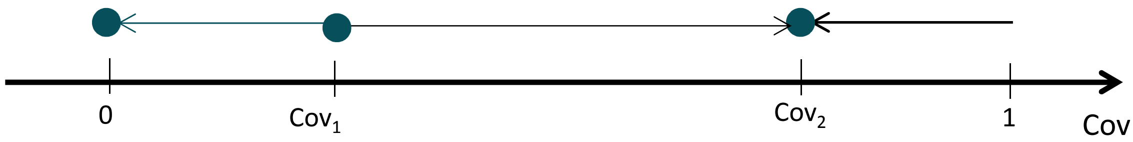


Figure 2.5: Evolution of CN coverage under wide distribution of CN users and the subscription fee f_s satisfies $\alpha < f_s < \frac{\beta^2}{4(\beta-\alpha)}$.

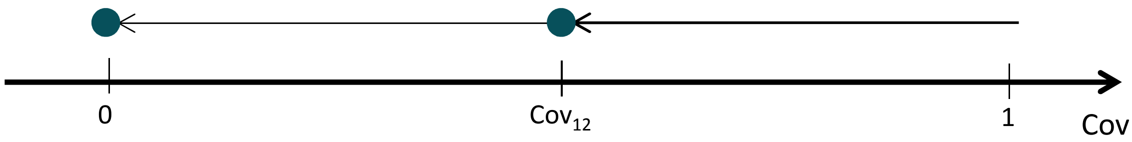


Figure 2.6: Evolution of CN coverage under wide distribution of CN users, in the special case that $f_s = \frac{\beta^2}{4(\beta-\alpha)}$.

2.4. Main findings and implications

To translate these results into hints for the CN, we should recall that this dynamic process has a starting point the coverage $Cov(0)$ at time $t = 0$. This is the result of the original investment made in the network by the team of volunteers who lead the CN initiative or by public/non-profit private authorities through some grant. The following can be concluded about the way the fee charged for subscription to the CN interacts with the initial investment made in it:

- Independent of how much effort/money is initially invested in the CN from community or public resources, the CN will end up without users if the pricing policy is too "aggressive". (Cases 1 and 5).
- Under narrow user distribution and for values of $f_s < \alpha$, there is a threshold value for the initial investment (resulting in $Cov(0) = Cov_1$), below which the CN cannot grow and dies out and beyond which, the CN achieves full coverage (Case 1). On the other hand, for each value of the initial investment, there is a maximum allowable fee the CNO can charge for subscription to the CN, to let it reach full coverage

$$f_s = Cov(0) \cdot (\beta - (\beta - \alpha) \cdot Cov(0)) < \alpha \tag{2.7}$$

- As a result, the maximum achievable revenue for the CNO under narrow user distribution is $N\alpha$, when f_s obtains its maximum allowable value $f_s = \alpha$.
- Under wide user distribution, the highest fee the CNO can charge without threatening the sustainable CN operation can increase to $\beta^2/4(\beta - \alpha)$. However, this may result in steady-state CN coverage values below full coverage.
- Indeed, the maximum CN revenue is achieved for non-full CN coverage and equals $\frac{4N}{27}\frac{\beta^3}{(\beta-\alpha)^2} - c_s$.

Although the precise quantitative and qualitative conclusions out of this analysis are closely related to the specific modeling assumptions (community members' decisions accounting for total coverage rather than coverage at places of interest to them, linear increase of coverage with the number of subscribers, uniformly distributed coverage factors a_u), they do capture the interrelation of the chosen subscription fee with the initial amount invested on the network and its subsequent coverage. In fact, a subset of the authors of [2] addressed some of these aspects in a follow-up work, focused on optimally setting the fee [8].

Key takeaway for CNs: The survival of a CN depends both on the initial investment made by the founders, and on a proper funding scheme based on individual contributions from the users of the network. These two factors are interrelated: the higher the initial investment, the higher the subscription fee that can be advertised and attract a critical mass that will foster the CN sustainability. The founding team should, therefore, weigh the resources it can invest in initiating the effort and adapt accordingly the financial support it will request from the rest of the community.

3. Building the network infrastructure up-front: the B4RN case

3.1. Motivation-inspiration

The other approach, far more rarely evidenced across the CN landscape, consists in building (almost) the whole network infrastructure up-front. Namely, contrary to the evolutionary CN growth described in chapter 2, a considerably larger investment is made on network infrastructure before the first users can be served. This is the case with fiber-based CNs such as the ones built by the Broadband for Rural North (B4RN). B4RN, registered as a non-profit community benefit society in UK, relies on the Fiber-to-the-Home (FTTH) technologies to provide high-speed Internet connectivity in rural Lancashire, in the north west of England.

The network deployment in a given area, of the size of a village or a civil parish, is carried out in response to an expression of interest from the respective community. B4RN openly advertises the cost and terms of service (connection and service fee, speed) and the community members state whether they would be willing to fund the network roll-out project [9]. The project costs relates to the roadworks (digging, or pole-mounting) for laying out fiber, which includes the required materials and the labor, as shown in Fig. 3.1.

When adequate interest is registered, a decision is made to undertake a project in the specific area. The project cost is actually funded by the community through the purchase of shares. B4RN issues shares of value that equals (or exceeds) the project cost under the provisions of a specific investment tool in UK, the HMRC Enterprise Investment Scheme (EIS), that favors the purchase of shares in riskier companies such as start-up companies. The issued shares are of value 1 GBP each, must be held for a minimum of 3 years, and can only ever be sold back to B4RN at par, namely they cannot be traded in a market. In return, the community members that buy shares become B4RN members with voting rights (one vote irrespective of the number of owned shares), can obtain 5% interest rate after the third year, enjoy tax relief of 30% thanks to the provisions of the EIS, and, when they invest on shares more than 1500 GBP, they save the connection fee worth 150 GBP. Shares are awarded in exchange for cash or labor, *i.e.*, community members can monetize effort they devote to digging or way rights through their farms.

The two main questions that need to be answered by the B4RN team every time a project for connecting a new village or parish is under discussion, are:

- (Q1) Does the interest of the community members suffice to cover the project cost, in this case, the offer of issued shares for the specific project?
- (Q2) If yes, how should this cost (*i.e.*, shares) be distributed among the community members?

Figure 3.1: Community involvement in laying out fiber through the farmland.

The first question relates to the feasibility and sustainability of the network deployment project. The question essentially is whether the value the community members attribute to the network connectivity exceeds the cost of the network infrastructure deployment. The second one touches on fairness aspects, *i.e.*, there should be some alignment between effort or money a member invests on the project and the benefit she obtains from it.

Currently, the B4RN team responds to the first question by considering the registered interest of the community. This interest, probably supported by personal exchanges through informational events, gives an indication about what the community members are willing to contribute to the project cost but leaves a lot up in the air. In the words of the B4RN's Director, Barry Forde, "this is something more than a gentleman's word". There is no guarantee that the cost will be eventually covered.

The approach to the second question is more pragmatic. The resources devoted by each member are translated to shares, which are investment tools. The more effort or money one devotes, the more shares she gets as reward, hence the tax relief is higher and so is the cumulative interest after the fourth year. The condition is that the project must be sustainable, otherwise the B4RN team retains the right to not distribute interest or do so at lower rate [9].

In what follows, we present an alternative formal and systematic way to respond to the two questions (Q1) and (Q2) which directly draws on the mechanism described in [3]. The goal of this analysis is to better understand the opportunities and the risks related to a model like the one adopted by B4RN, and suggest ways to potentially improve it.

3.2. A cost-sharing mechanism for B4RN's community-led projects

Let c be the cost of the deploying the fiber network to the parish under question, as estimated by the B4RN team, N the set of properties with $n = |N|$, and $b_1, b_2, .., b_n$ what the community members actually intend to invest in the network deployment project. The mechanism proceeds in two stages.

Stage 1: The community members are invited to individually state their estimates, $v_i, i \in N$ for the total investment (in cash, effort or both) the whole community is willing to invest on shares. Say, without loss of generality, that $v = v_i*$ is the maximum of these estimates and $i*$ the member who submits it. Then,

- if $v < c$, the project is not carried out.
- otherwise, the mechanism proceeds in stage 2.

Stage 2: The community members submit, again individually, bids $\beta_i > 0$, representing the investment each intends to make in the fiber network. In general, a bid β_i may differ from the actually intended investment b_i because member i expects that this way, she can save on what she will be asked to pay in the end. There are three scenarios:

- If $\beta_N = \sum_{i \in N} \beta_i \geq v$, the project is carried out and the members pay proportionally to their bids β_i. More specifically, given:

$$\gamma(\beta_i; \beta_1, \beta_2, .., \beta_{i-1}, \beta_{i+1}, ..\beta_n) \equiv \gamma(\beta_i; \beta_{-i}) = \frac{\beta_i}{\beta_i + \beta_{-i}} c = \frac{\beta_i}{\beta_N} c \tag{3.1}$$

  the cost shares $\{x_i\}, i \in N$ for each community member are given by:

$$\begin{aligned} x_i &= \gamma(v - \beta_{N \setminus i}; \beta_{-i}) \quad i \neq i^* \\ x_i^* &= c - \sum_{j \neq i^*} x_j \end{aligned} \tag{3.2}$$

- If $\beta_N < v$, the project is not carried out and the member i^*, who made the highest statement v in the first stage, pays "compensation" fees to each other member i.

$$r_i = v - \beta_{N\setminus i} - \gamma(v - \beta_{N\setminus i}, \beta_{-i}) \tag{3.3}$$

Intuitively, this is the penalty she pays for trying to artificially increase the value of the project for the community (see also toy-example 2 in section 3.4).

3.3. Properties of the mechanism

The mechanism features the following desirable properties:

- It is truthful. The equilibrium (Nash Equilibrium) strategy of the community members is to submit their true anticipation about what the community is willing to invest as a whole in the project in the 1st stage ($v_i = \sum_{i\in N} b_i$) and the true amounts they are ready to invest individually at the 2nd stage ($\beta_i = b_i$).
- It makes the right decisions. As a result of the first property, the mechanism decides to go on with the deployment of the network infrastructure as long as the community members indeed intend to cover its cost. When this is the case, they share the cost according to their true intended investment (in proportion to them or in line with some other function), which reflects the value the fiber network bears for them.
- It is budget-balanced. The amounts invested by the community members sum up exactly to the project cost c.

The following toy examples further clarify these properties.

3.4. Coping with misbehaviors: toy-examples

In coping with questions (Q1) and (Q2), the B4RN association relies on the statements made by the community members about the value they expect to derive out of the project. In general, these statements are not guaranteed to be sincere. Depending on how these statements relate to the cost share each member undertakes, namely the cost-sharing mechanism, individual members may have reasons to declare deliberately inflated or deflated benefits, in an attempt to influence the decision about the project and reduce their cost share. In what follows, we demonstrate how the truthful property of the mechanism discourages certain expressions of strategically falsified statements.

In all subsequent examples, assume that there are $n = 3$ members and the intended investments by the three members are $b_1 = 8, b_2 = 5$ and $b_3 = 2$.

3.4.1. Example 1: Truthful statements (equilibrium), $c = 10$

At the equilibrium, the three members would submit at the first stage estimates $v_1 = v_2 = v_3 = 15$. Any of the three could be considered as the winning bid at 1st stage; say that $i^* = 1$. Since $v = 15 > c$, the provisional decision, subject to the stage 2 outcome, is that the project can be carried out.

At the stage 2, the bids of the three members equal their intended investments, $\beta_i = b_i$. Hence, $\beta_N = v$ and the project is indeed carried out. The cost shares of the three members are: Member 2 invests

$$x_{2,eq} = \frac{v - \beta_1 - \beta_3}{v} c = (15 - 8 - 2)c/15 = 5c/15 = c/3 = 3.33 \tag{3.4}$$

Member 3 invests

$$x_{3,eq} = \frac{v - \beta_1 - \beta_2}{v} c = (15 - 8 - 5)c/15 = 2c/15 = 1.33 \tag{3.5}$$

and member 1 covers the residual cost

$$x_{1,eq} = c - x_{2,eq} - x_{3,eq} = c–c/3–2c/15 = (15 - 5 - 2)c/15 = 5.33 \tag{3.6}$$

This is the nominal case, where members act rationally in the Nash equilibrium sense, *i.e.*, they consider strategically their alternatives and end up with statements (bids) that no one would unilaterally want to deviate from. The value of issued shares is oversubscribed by the community members and members end up investing amounts that are significantly less than and in proportion to their intended investments.

3.4.2. Example 2: Cheat by trying to force the project execution, $c = 16$

Assume now that one of the members, say member 1, lies about the estimate of the overall intended investment from the community, in an attempt to ensure that the project is carried out. Note that due to the way the mechanism works, an overestimate at the first stage needs to be combined by an analogously inflated statement of her own intended investment at stage 2. For instance, let $v_1 = 16$, $v_2 = 15$ and $v_3 = 15$ and $\beta_1 = 10$, $\beta_2 = 5$, $\beta_3 = 2$, *i.e.*, the other two members truthfully state their intended investments.

The winning estimate at stage 1 is $v = 16$ by member 1. Since $v \geq c$, the outcome of stage 1 is positive for the fiber network project. Then, at stage 2, the sum of bids is $\beta_N = 17 > v$, hence the project is indeed carried out and the members are asked to invest the following amounts: Member 2 needs to invest

$$x_2 = \frac{v - \beta_1 - \beta_3}{v} c = (16 - 10 - 2)c/16 = 4c/16 = c/4 = 4 < b_2 \tag{3.7}$$

Member 3 needs to invest

$$x_3 = \frac{v - \beta_1 - \beta_2}{v} c = (16 - 10 - 5)c/16 = 1c/16 = 1 < b_3 \tag{3.8}$$

whereas the cheating member has to invest

$$x_1 = c - x_2 - x_3 = 16–4c/16–c/16 = (16 - 4 - 1)c/16 = 11 > \beta_1 > b_1 \tag{3.9}$$

In this case, the member who cheats and forces the project to happen, although the intended investment by the community does not suffice, undertakes a significant additional cost, exceeding what she actually intended to invest in the network. This cost is offloaded by the other two members who honestly state their intended investments.

Note that if the cost was $c = 10$, as in the first example, and member 1 follows the same cheating strategy ($v_1 = 16, \beta_1 = 10$), possibly with the intention to secure the execution of the project, the resulting investments at the second stage would be:

$$x_3 = 2.5 < x_{3,eq}$$
$$x_2 = 0.625 < x_{2,eq}$$
$$x_1 = 6.875 > x_{1,eq}$$

Hence, the cheating member is punished by having to invest a higher share than what would arise if her bid was truthful. The other two members, who truthfully state their investment intentions, are “rewarded” with lower cost shares.

Even worse, under the same scenario and slightly higher but investment-covered cost, $c = 12$, the resulting investments at the second stage would be:

$$x_3 = 2.82$$
$$x_2 = 0.71$$
$$x_1 = 8.47 > b_1$$

so that the cheating member not only undertakes a higher part of the cost than at the equilibrium (truthful bids) but also needs to invest more than what was her original intention ($b_1 = 8$).

3.4.3. Example 3: Cheat by trying to reduce the individual cost share, $c = 10$

Assume that all members submit $v_i = 15$ at the first stage but the winning member at stage 1, say $i^* = 1$, underbids at stage 2, say $\beta_1 = 6$, in a (naive) attempt to lower her investment share. Obviously, the project will not be carried out since $\beta_N = 13 < v$. Nevertheless, the cheating member will be penalized with rewarding payments to the other two members, that is member 1 pays member 2

$$r_2 = 2.33 \tag{3.10}$$

and member 3

$$r_3 = 1.33 \tag{3.11}$$

namely, a total of $r_2 + r_3 = 3.66$. This way, the mechanism discourages attempts of members to reduce their cost shares after securing that it goes through the first stage.

3.5. Practical issues

With respect to the current state of affairs, the mechanism introduces some complexity in the process of determining the cost shares each community member should undertake.

First, effort is required to explain the rules to all community members. Computationally, the involved maths are trivial; however, it needs some more time to grasp the rationale of the mechanism and be convinced about its nice properties (*e.g.*, truthfulness).

It then requires some process through which the community members can submit their bids as well as some provisions to ensure that the process runs smoothly. Although the process is automated, there should be some trust to the entity supervising the process. To this end, it would be probably easy to elect a committee for the purpose. Anyway, the community bonds and face-to-face relationships with the B4RN team rather ease the task.

Finally, on the input side, the members should have some estimate about what the overall community is ready to invest in this project. Again, the community dimension helps here. Rough estimates of this kind could result out of discussions and chats between the community members, the way these naturally happen daily in the life of the community.

In any case, the main advantage of the mechanism is that it can indicate whether a project is sustainable or not and come up with a systematic way to split the costs, while discouraging possible intentions of individuals to free ride and save their own cost share. Given that CNs have a good record of trying out new ideas, structures, technologies, an experimentation with the mechanism would be worth.

Key takeaway for CNs: CN initiatives that have to invest large amounts of money up front, as the case is with fiber-cable CNs, need to have a good understanding of the community's intention to support the project and secure its commitment to it. Mechanisms that can serve this purpose, such as the one analyzed in this chapter, are available in economics and it is important for the founding teams of these CNs to familiarize with them and exploit them.

4. Novel subscription schemes: the Sarantaporo.gr case

4.1. Motivation - inspiration

In end of 2016, the Sarantaporo.gr CN [1] decided to revise its funding model and implement a different approach, which is already in effect since the beginning of 2017. The motivation for such a decision lay in the non-sustainability of their funding sources. Table 4.1 lists what have been the main funding sources for the Sarantaporo.gr CN in year 2016 and how they compare with each other as revenue sources for the network.

Table 4.1: Mix of revenue sources for the Sarantaporo.gr CN.

Source of Revenue (2016)	Percentage
Grants/research programs	63.7%
Donations	10%
Member subscriptions	23.7%
Services	2.2%

Table 4.1 demonstrates heavy dependence of the CN on European Union (EU) and national social cohesion programs, which account for roughly two thirds of the CN's funding. A 10% of the network revenues has been contributed by occasional one-time donations, collected primarily through a crowdfunding campaign and the disposition of promotional material about the CN (*e.g.*, T-shirts). On the other hand, the contribution of community member subscriptions is small, both in absolute and relative figures, accounting for less than one quarter of the network revenues. In fact, the subscriptions were annual and at village level, namely, each one of the 14 villages covered by the CN had to collect from its residents the subscription amount through a crowdfunding process. Responsible for running the crowdfunding was either some association (*e.g.*, the local cultural association) or a person who was nominated for this purpose by the Sarantaporo.gr core team.

By the end of 2016 the members of Sarantaporo.gr acknowledged two things: a) acquiring funding through EU and local programs requires a lot of resources and is very time-consuming, and b) the village-oriented subscription scheme didn't work. More specifically, the collection of the subscription fee depended mainly on whether a local opinion leader was involved, who was able to organize the village's *yearly crowdfunding* for the network. When such a person was involved, the crowdfunding would be successful. In most cases, though, no such people were available. There was even a case, where the collaboration with a local cultural association resulted in friction and conflict: the association kept the largest part of the collected funds for its own operation despite advertising that the funds were collected for the community network. These events raised concerns for the economic sustainability of the network.

One alternative would be to introduce mandatory individual fixed-price subscriptions for anyone using the CN. However, there are two issues with individual member subscriptions. First, more often than not, these subscriptions are optional for the community members, to highlight the participatory and non-profit characteristics of the initiative. As a result, high levels of free riding are evidenced (much as the case is with the village-oriented subscription of Sarantaporo.gr): many members use the network without entering any subscription relationship with it, thus not contributing at all to its operational costs. Secondly, in CNs it is of highest priority to ensure affordable network access to all community members, including the financially weakest ones. With the default fixed price subscription scheme, this is only possible with very low fees that end up shrinking what CNs can reap from what is meant to be their primary funding source.

Hence, the team decided to launch and experiment with a novel subscriptions scheme, which we hereafter call *collective subscriptions*.Under collective subscriptions, the elementary subscribed unit is a *CN node*, rather than an individual or household using the CN services, and the node subscription fee is shared between those users who subscribe to it.

Besides matching well the participatory and sharing ideals of CNs, collective subscriptions are a tool that can better mobilize the community and create awareness in it about the need to sustainably fund the CN. Our analysis of the scheme suggests that it outperforms the default fixed-price subscription model with respect to both objectives, combining higher subscription revenue with increased community inclusion. The scheme essentially serves as a countermeasure against free riding, motivating existing CN users to actively recruit new subscribers as a means to reduce their own share of the subscription fee. At the same time, they help match better the different motivations of people for using the CN: the more tech-savvy people are more incentivized to create and operate nodes, whereas people that are primarily interested in Internet access may contribute economically to its growth.

In the remainder of this chapter, we summarize our analysis of collective subscriptions and our findings giving brief accounts of the system model, the problem formulations that emerge, their solutions, and the take-aways for those teams that may want to consider collective subscriptions as a subscription mechanism for their own CN. The technical report with the full details of the analysis, which is itself an extended version of a paper under double blind review, is attached to the deliverable as Appendix I.

4.2. System model

We consider a community network (CN) providing network connectivity to a community of users $\mathcal{U}$, with $|\mathcal{U}| = U$, through a set of $\mathcal{N}$ nodes, with $N = |\mathcal{N}|$. The original deployment of these nodes is typically funded by volunteers-members of the community, sometimes with sponsoring by public agencies. Collective subscriptions are a funding tool aiming at the monthly operational expenses of the nodes.

The main actors in this setting are:

4.2.1. The CN operator

The term denotes the small group of people who have initiated and typically operate the CN. They are often organized as a non-profit entity and one of their main tasks is to ensure the sustainable funding of the effort. The Community Network Operators (CNOs) leases network access capacity from commercial upstream ISPs. It then determines the (monthly) node-level subscription fee f_s that will make up for the cost of Internet transit and the operational expenses of the network, but also fund the further growth of the CN.

4.2.2. The CN users

These are community members who can potentially use the CN after subscribing to it[1]. Each CN user $u \in \mathcal{U}$ has the option to join the collective subscription of one node. The nodes of interest to her form a user-specific subset $\mathcal{N}_u \subseteq \mathcal{N}$, which typically includes those nodes the user tends to use more often such as those in the vicinity of her residence or workplace (ref. Fig. 4.1).

Users appreciate differently the value of the CN and Internet connectivity and this is directly reflected to the maximum monthly price pcl_u, $u \in \mathcal{U}$ they are willing to pay for participating in it. These personal *price ceilings* could become known to the CNO through questionnaires or informal communication exchanges taking place

[1]Hereafter, the term user denotes the holder of individual subscriptions, *i.e.*, a user may correspond to a household, as with typical subscriptions to commercial network operators.

Figure 4.1: Example physical system layout and options available to CN users under the collective subscriptions' scheme. Each user may join the collective subscription of one node out of a subset of the N CN nodes or abstain (*i.e.*, "join node n_0").

within the community. Let the CN users be indexed in order of decreasing price ceilings so that

$$pcl_i \leq pcl_j \; \forall j < i \tag{4.1}$$

Under collective subscriptions *the pricing at node level is uniform across nodes*, *i.e.*, the CNO does not discriminate between different CN nodes when setting f_s. On the other hand, *the subscription share f_s/k individual CN users end up paying is not the same for every user* but rather depends on the number k of users who join a node's subscription. This way, the benefits of the scheme are twofold. First, since the individual fee decreases with the number of users subscribing to a given node, existing CN users are given a tangible incentive to actively recruit new users to the CN. Secondly, as we show in section 4.4, the scheme can significantly increase the revenue from subscriptions when compared to its standard fixed-price individual subscription counterpart.

Our analysis of collective subscriptions addresses both possible modes of applying them to CNs. In the first mode, collective subscriptions are centrally coordinated by the team operating the CN (*CN operator-CNO*). This suits better small communities with typically tighter links and personal contact between the community members and the team of people, usually members of the same community, who run the CN. In such settings, users are more willing to trust and adopt the assignments, as determined by the CNO. The joint assignment of CN users to node subscriptions and determination of the common CN node subscription fee by the CNO (in the case study, by the Sarantaporo.gr core team), gives rise to an optimization problem (*collective subscription problem*) with a non-trivial objective function.

As the size of the community grows, the motives and aspirations of users joining a CN vary broadly [10]. Many users are primarily attracted by the possibility to get cheap Internet access. In those CN instances, more relevant becomes the second mode of applying collective subscriptions, which is in line with current pricing practices in letting users have the final say. In this second mode, the process evolves in two steps. First, the CNO announces the common node-level subscription fee and, then, the community members decide whether to join a node subscription and which one so as to minimize their subscription fee share. Here, the two parties may strategize, aiming at maximum revenue and community inclusion (CN operator) and minimum subscription share (community members) and, thus, motivating a two-level strategic game (*collective subscriptions game*).

4.3. Analysis

4.3.1. The *collective subscriptions problem*

Consider a feasible partition $p = (p_0, p_1, p_2, .., p_N)$ of CN users to CN node subscriptions, $k_n = |p_n|$ being the number of users who share the subscription of node $n \in \mathcal{N}$, and k_0 the number of users who abstain from the CN. To ease notation, we introduce a virtual node n_0 so that p_0 (k_0) is the subset (number) of users who join n_0, and we can define the extended node sets $\mathcal{N}_u^+ = \mathcal{N}_u \cup n_0$, for each CN user, and $\mathcal{N}^+ = \mathcal{N} \cup n_0$, for the overall CN.

Given this partition, the maximum fee the CN operator can collect from node n is

$$fee(n) = k_n \cdot \min_{u \in p_n} pcl_u \tag{4.2}$$

that is, for a given subset of users that share a node's subscription fee, the per-user fee share cannot exceed what the user with the minimum price ceiling is willing to pay.

At network level, since the subscription fee is uniform across all CN nodes that attract subscribers, the CNO can gather a fee that equals

$$F_{CNO}(p) = \min_{\substack{n \in \mathcal{N} \\ k_n > 0}} (n) \cdot \sum_{n \in \mathcal{N}} \mathbb{1}_{\mathbb{Z}^+}(k_n) \tag{4.3}$$

where $\mathbb{1}_{\mathcal{A}}(x) = 1$, for $x \in \mathcal{A}$, is the indicator function and $\mathbb{Z}^+$ is the set of positive integers. Overall, the revenue the CNO can collect is subject to a $double\ min\ effect$: first, what can be collected at node level is determined by the user with the minimum price ceiling, in line with (4.2); then, at network level, the common for all nodes collective subscription fee is determined by the minimum fee that can be collected across all nodes, in line with (4.3).

Hence, the partition of CN users that maximizes $F_{CNO}(p)$ is the solution to the optimization problem

$$\begin{aligned} \max_{p} \quad & F_{CNO}(p) \\ s.t. \quad & k_n = \sum_{u : n \in \mathcal{N}_u} x_{un} \; \forall n \in \mathcal{N}^+ \; (OPT) \\ & \sum_{n \in \mathcal{N}_u^+} x_{un} = 1 \; \forall u \in \mathcal{U} \\ & x_{un} \in \{0, 1\} \; u \in \mathcal{U}, n \in \mathcal{N}^+ \end{aligned} \tag{4.4}$$

where $x_{un} = 1$ when user u shares the subscription fee for node n and $x_{un} = 0$ otherwise.

If the per node collected fee in (4.2) equaled the sum of subscribers' price ceilings rather than their minimum, we would face an instance of the restricted max-min fair allocation problem (see, for example, [11]). It is the *min* operator in (4.2) that renders the objective function in (OPT) non-trivial.

The generic optimization problem can be simplified in two ways: (a) with respect to the subscription assignment preferences, *i.e.*, $\mathcal{N}_u = \mathcal{N}, \forall u \in \mathcal{U}$; (b) with regard to the price ceilings, *i.e.*, $pcl_u = pcl\ \forall u \in \mathcal{U}$. In the attached technical report, we consider in detail the three cases that emerge when (a) and (b) hold either separately or simultaneously.

4.3.2. The *collective subscriptions game*

The game model for the collective subscriptions comes under the broader category of the leader-follower dynamic games. The leader is the CNO that determines and announces the common node-level subscription fee f_s to all CN nodes. The followers are the CN users who respond to the advertised fee by selecting a node to subscribe to the CN or abstaining from it. Hence, for each distinct fee value announced by the CNO, a proper subgame (stage II game) is played by the U CN users.

4.3.2.1. Stage II: the CN users' subgame

For a given fee choice f_s by the CNO, the CN users' responses are modeled by the strategic-form game $G_N(f_s) = \langle \mathcal{U}, (\mathcal{S}_u)_{u \in \mathcal{U}}, (\pi_u)_{u \in \mathcal{U}} \rangle$ where:

- $\mathcal{U}$ is the addressed community, *i.e.*, the set of potential CN subscribers;

- $\mathcal{S}_u = \mathcal{N}_u \cup N_0 \subseteq \mathcal{N}'$ is the set of singleton strategies of user u, with $\mathcal{S}_a \neq \mathcal{S}_b$, for $a, b \in \mathcal{U}$, in general (ref. section 4.2); and
- π_u are the user-specific payoff functions, which for $n \in \mathcal{N}$ depend on the number of users subscribing to n, k_n, and their price ceilings

$$\pi_u(n, s_{-u}) = pcl_u - \frac{f_s}{k_n} \tag{4.5}$$

  whereas they are zero for n_0, $\pi_u(n_0, s_{-u}) = 0$, s_{-u} being the set of actions of all other users other than u.

Depending on the action profile $\mathbf{s} = (s_1, s_2, ..., s_U)$, *i.e.*, the combination of strategies the CN users follow, their distribution across the N+1 nodes forms a partition $Q(\mathbf{s}) = (q_n(\mathbf{s}))_{n \in \mathcal{N}'}$, with

$$q_n(\mathbf{s}) = \{u \in \mathcal{U} | s_u = n\} = k_n \tag{4.6}$$

In the technical report, we prove the following Nash Equilibrium (NE) existence result for $G_N(f_s)$:

Proposition 1. *The game $G_N(f_s)$ possesses at least one pure-strategy NE.*

The proposition states that there is at least one combination of actions by the CN users, *i.e.*, their joining specific collective subscriptions, that forms a NE (a state of the game, in which every CN user has nothing to gain by unilaterally changing subscription). The proof involves showing that the game $G_N(f_s)$ belongs to a particular family of games called exact potential games [12]. We build on this result, to devise a search process for NE. Exact potential games possess the Finite Improvement Property (FIP): it is possible to start from an arbitrary initial action profile and reach the NE action profile, through a finite number of myopic steps, where each time a single user changes action (here: subscription she joins) so that her payoff is increased. By repeating this process multiple times, each time starting from a different random initial action profile, we can arrive at different NE.

In the report, we also show that the derivation of the game NE profiles is simplified enormously under symmetry in either the CN users' strategy sets ($\mathcal{S}_u \equiv \mathcal{N}', \forall u \in \mathcal{U}$) or their payoffs ($pcl_u = pcl_0 \ \forall \ u \in \mathcal{U}$)

4.3.2.2. Stage I: the fee choice by CNO

Let $\mathcal{Q}_{NE}(f_s)$ be the set of CN user partitions into CN nodes at the NE action profiles of users under subscription fee equal to f_s. Each partition $Q(s) \in \mathcal{Q}_{NE}(f_s)$ uniquely determines:

- the number of users $|q_n(s)|$ that share the subscription of node n, see (4.6), and the number of CN nodes that attract at least one CN user who can pay the subscription fee

$$N_{sub}(Q) = |\{n \in \mathcal{N} \mid |q_n(s)| > 0\}| \tag{4.7}$$

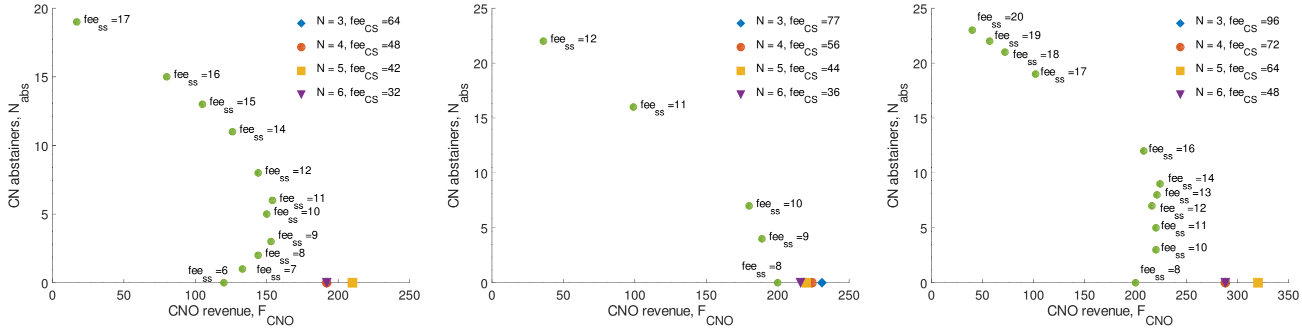
- the number of users who have not joined any CN node and abstain from the CN

$$U_{abs}(Q) = |q_{n_0}(s)| \tag{4.8}$$

The CNO will then announce a fee

$$f_s^* = arg \max_{f_s, Q \in \mathcal{Q}_{NE}(f_s)} \left(f_s \cdot N_{sub}(Q)\right) \tag{4.9}$$

that maximizes its revenue.



a. $U = 20, pcl \sim \mathcal{U}\{5, 17\}$ b. $U = 25, pcl \sim \mathcal{U}\{8, 12\}$ c. $U = 25, pcl \sim \mathcal{U}\{8, 20\}$

Figure 4.2: CNO revenue and CN abstainers under fixed price individual subscriptions (subscript "ss") and collective subscriptions (subscript "cs"). Some of the markers that are listed in the plot legends for collective subscriptions are not visible because they overlap with each other.

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4.4. Main findings and implications

The two main performance metrics of interest regarding the collective subscriptions are the revenue, F_{CNO}, that the CNO can extract from the scheme and the resulting number of CN abstainers, U_{abs}. In general, the two metrics depend on (a) the number of CN nodes and their arrangement in physical space; (b) the number of CN users and their mobility patterns within the CN coverage; (c) the amounts of money that CN users are willing to pay for CN connectivity (*i.e.*, price ceilings). We assess the impact of these factors through simulations with both real and synthetically generated data. The real data refer to actual topologies of the CNs in the Sarantaporo.gr area and are used to infer the subscription assignment preferences of users. With synthetic data, we consider different numbers of CN nodes and users and let the number of node subscription alternatives per user, $(|\mathcal{S}_u|)_{u \in \mathcal{U}}$, as well as their price ceilings, $(pcl_u)_{u \in \mathcal{U}}$ vary stochastically.

The main take-aways from the evaluation process are summarized in the following paragraphs.

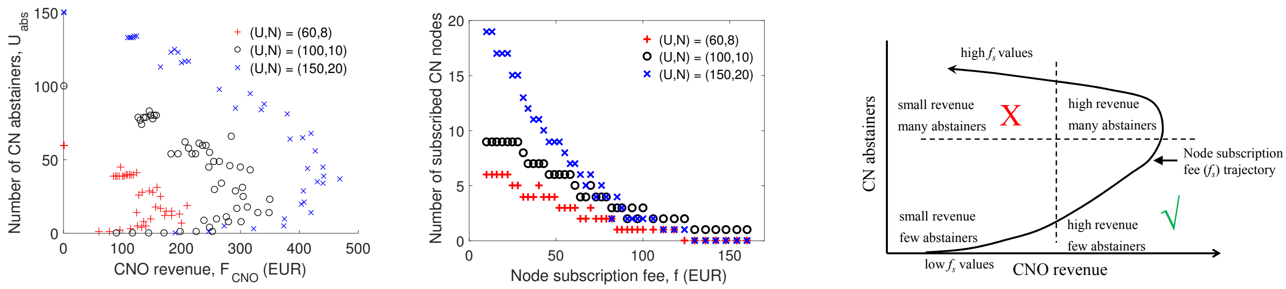
Collective subscriptions *vs.* fixed-price individual subscriptions:

The plots in Fig. 4.2 were generated using the data describing nodes and users we extracted from the Sarantaporo.gr network. They are representative of the way optimized collective subscriptions compare with fixed price individual subscriptions. In the second case, the only way to make the CN access affordable to the whole community is by setting the fee to the minimum price ceiling across all users. A higher revenue may be feasible by choosing higher fees but this comes at the expense of more abstainers and lower community engagement. Increasing the fee even further is not an option since most of the community cannot afford it and the revenue collapses.

On the contrary, with collective subscriptions, the CNO can achieve (F_{CNO}, N_{abs}) values (the markers at the right bottom end of plots in Fig. 4.2) that dominate those achievable with individual subscriptions. The scheme groups properly CN users into subscriptions and adapts the per node fee to the number of nodes maximizing both the CNO revenue and the community engagement. Essentially, the collective subscription scheme emerges as an indirect way to apply mild price discrimination and extract from CN users fees in line with the value they obtain from the CN.

In the attached technical report, we prove that when $(U - N_{abs}) = \delta \cdot N, \delta \in \mathbb{Z}^+$

Proposition 2. *For any given set of users and their corresponding price ceilings, the collective subscription scheme can yield* (F_{CNO}, N_{abs}) *values that Pareto-dominate those obtained with fixed price individual subscriptions.*



a. F_{CNO} vs. U_{abs} b. Subscribed CN nodes c. Variation with node subscription fee

Figure 4.3: Variation of CNO revenue, CN abstainers and number of subscribed CN nodes with the CN node subscription fee: $|\mathcal{S}_u| \sim \mathcal{U}[1,3]$

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Revenue *vs.* social inclusion - tuning the scheme:

Instructive about the CNO's fee setting strategy is Fig. 4.3. The scatter plot in Fig. 4.3a reveals clear patterns in the evolution of the CNO revenue and CN abstainers' number as the subscription fee increases, which persist across different (U,N) value pairs. As f_s grows, the CNO revenue-CN abstainers space is traversed in the way shown in Fig. 4.3c. Favorable f_s values are those producing value pairs at the bottom-right part of the space, where high CNO revenue is combined with few CN abstainers. On the contrary, f_s values inducing NE (F_{CNO}-N_{abs}) pairs at the top-left quarter of the space should be avoided since they result in a combination of many unfavorable values: high number of CN abstainers, capability to support only very few collective subscriptions (Fig. 4.3b), and low subscription revenue.

Further insights to the performance of collective subscriptions and their sensitivity to different parameters can be found in the attached technical report.

Key takeaway for CNs: The accumulated experience with CNs suggests that their main and most reliable funding source are the contributions of its own community members. To this end, a collective subscription scheme is a more attractive option than an individual fixed-rate subscription scheme. Collective subscriptions, as applied in the Saantaporo.gr CN, allow more community members to join the CN, serving as a countermeasure to free riding, and increase its funding, thus ensuring a higher amount of money for its growth and smooth operation. The analysis in this chapter can serve as a guide for tuning the scheme to specific CN instances.

5. Engaging the for-profit private sector: the guifi.net case

The research work on the guifi.net network is mainly inspired by its business model, a pioneering model across CNs worldwide regarding its provisions for the participation of for-profit entities as stakeholders in the CN infrastructure.

5.1. Background

As reference for our work stands the open access model for network infrastructure and services in Fig. 5.1[1]. In what follows, we present the main ingredients of this model in more detail.

5.1.1. Network layers

According to the concept of the model, the network functionality can be structured into three distinct but inter-dependent layers: a) *passive infrastructure*, b) *active infrastructure* and c) *services*.

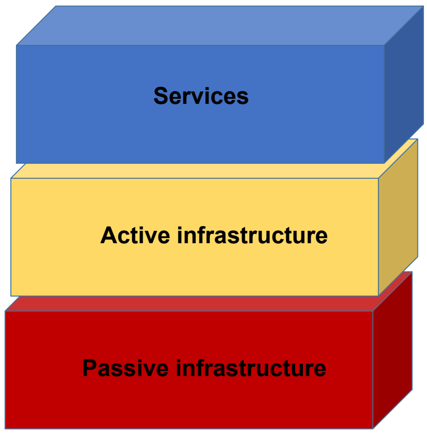


Figure 5.1: The layers of a broadband network

The *passive infrastructure layer* includes physical infrastructure that depends on the link technology in use, *e.g.*, fiber, copper, radio. Ducts, cables, masts, towers and other equipment of non-electronic nature, with lifetime in the order of decades, are part of this infrastructure layer. Its development typically demands high Capital Expenses (CAPEX) and does not favor frequent upgrades. However, its operational expenses Operational Expenses (OPEX) are relatively low.

[1]LLUB (or LLU) in 5.1 stands for Local-Loop Unbundling: the regulatory provision that multiple operators can use, as lawful beneficiaries, the local loop, that is the physical wire connection between the local exchange and the customer typically that is under the ownership of one operator.

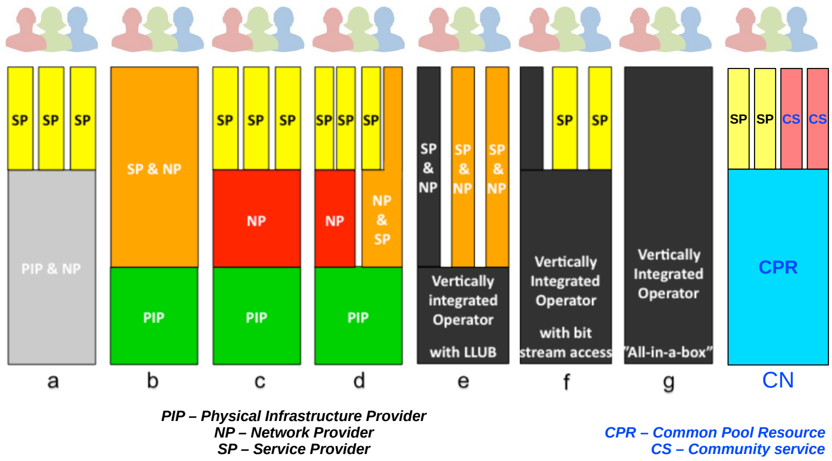


Figure 5.2: The layers of a broadband network (adapted from [4]).

The *active infrastructure* layer denotes electronic physical equipment such as routers, switches, antennas, transponders, control and management servers. The OPEX of the active equipment (*e.g.*, electricity costs) is high but its capital expenditure is usually low. The active equipment needs to follow the advances of technology and get renewed frequently, *i.e.*, more than once within a decade.

Finally, the *service* layer corresponds to the telecommunication services provided on top of the passive and active infrastructure. Examples include Internet access, telephony (*e.g.*, VoIP), access to media content (television, radio, movies). The two infrastructure layers serve as prerequisite for the service layer.

5.1.2. Business actors and possible roles

The business actors are typically providers of network equipment and/or services, assuming roles at one or more of the three network layers in Fig. 5.1. Telecom operators and private companies, public authorities, but also local cooperatives and housing associations may count as business actors.

The *Physical Infrastructure Provider (PIP)* owns the passive equipment and undertakes equipment maintenance and operation responsibilities. Depending on which network parts they possess, PIPs can be divided into *backbone PIPs* and *access area PIPs*. Backbone PIPs invest in the backbone network infrastructure, while access area PIPs own and moderate the infrastructure aimed for providing connections to the end users, *i.e.*, first-mile connectivity.

The *Network Provider (NP)* owns and operates the active equipment. It leases physical infrastructure installations from the PIPs and makes its equipment available for the provision of services by Service Providers (SPs). Network providers may be public authorities, private companies, local cooperatives who own the equipment or entities who are subcontracted to operate them by one of the aforementioned owner entities.

The *Service Providers (SPs)* offer services within the network. They are typically for-profit companies that utilize the network's active and passive equipment to offer their services to end users in exchange for monetary compensation. They need access to the NP's interface and install their own devices if and where needed.

5.1.3. Open business models

Figure 5.2 captures the possible models that can emerge with respect to the functional separation across the three layers. The models from *a* to *g* are generic models used in the market, the CN model is the one used by guifi. Although the borderline between the respective actors is not always clearcut, they range all the way from variants of vertical integration in *e, f, g*, to partial role separation in *a, b, d*, and full functional separation in *c*. The models imply different alternatives and therefore competition at each layer, except for the passive infrastructure, where a single actor is typically in charge of deploying and operating it. Whereas all models except for *g* offer alternatives with respect to service provision, only*d, e* provide alternatives regarding network provision.

An example of case (d) is the municipal fiber cable network of Stockholm (Stokab), where a municipality-owned company undertakes the role of PIP and multiple NPs may access its dark fiber infrastructure [4]. Almost all instances of Mobile Virtual Network Operators (MVNOs) are examples of case (e), where the MVNO leases a small part of the network resources of a Mobile Network Operator (MNO). Crowdsourced community network infrastructures, which are the focus of this work, represent a distinct case in Fig. 5.2 (denoted as CN), where the network infrastructure, both passive and active, are a community resource and service providers may deliver services on top of it under a commons license. This is the case of the guifi.net CN [6] that we explore further in what follows.

More specifically, we consider the roles the CN does or could undertake in two concrete scenarios of service provision. In the first one, the (guifi.net) CN serves as a provider of passive and active network infrastructure, which can be used by multiple Internet access service providers (SPs) to deliver services to the end users. This is a real scenario, currently in operation. In the second scenario, the CN deploys storage and computation infrastructure, what is referred to as community clouds in [13], which is available to different SPs for delivering services to end users. This is a possible scenario for the operation of local community clouds in a sustainable manner.

The two scenarios are in some ways similar and in some others complementary. In both cases, the CN acts as an intermediary between end users and service providers, taking care of the infrastructure that enables the latter to serve the former. This infrastructure is maintained and managed with the prescriptions of the theory of commons [14]. On the other hand, in the second scenario, SPs are explicitly involved in funding the cloud infrastructure and there is much higher granularity in the way this infrastructure can be organized into slices. This favors richer pricing policies beyond the flat schemes that are typical in Internet access service. It is worth noting that as software defined networking advances, the two scenarios will tend to converge, with *network slices* including also shares of network links and radio network resources, beyond virtual machines, disk space, and processor cycles.

5.2. Network infrastructure sharing for Internet access

5.2.1. System model

The model we consider is outlined in Fig. 5.3. The three main types of actors are the end users, the network infrastructure provider (NIP) and the multiple service providers (SPs). One important element is that for a commercial ISP the "service providers" are entities that make service available on top of the Internet access provided by the ISP. In the Guifi CN model instead, the passive and active infrastructure are treated as generic assets that can be used to provide services, among which, Internet access. In the rest of this analysis indeed, an SP generally refers to an entity that provides Internet access to people, using the active and passive infrastructure made available as Common Pool Resources (CPR).

5.2.1.1. Network Infrastructure Provider

The NIP entity combines the roles of PIP and NP in Fig. 5.2. It is responsible for deploying the network infrastructure, both passive and active. When the network is a community network, it corresponds to the small group of people who have initiated and typically operate the CN. They are often organized as a non-profit entity and one of their main tasks is to ensure the sustainable funding of the effort.

From the economic point of view, the NIP initially invests an amount c_0 in the network. This is used to purchase equipment and set up the infrastructure, both passive and active, including also labor expenses, *e.g.*, for digging

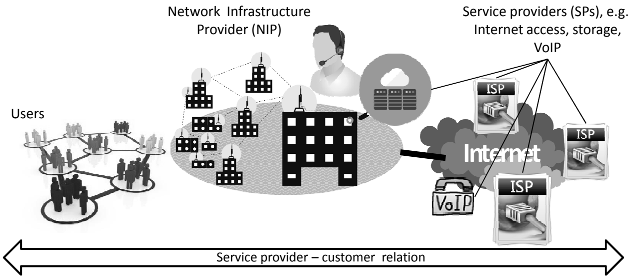


Figure 5.3: The actors in shared infrastructures model.

tasks. This investment lets the NP cover a particular geographical area. The actual geographical coverage of the deployed network is a non-decreasing function $I(c_0)$ of the invested amount and relates directly to the number of users N the network can reach (*market size*); namely, $N = f(I)$, where $f(\cdot)$ a non-decreasing function of network coverage. For model simplicity, we hereafter reduce the dependence of the customer base N on the investment c_0 to a single non-decreasing function

$$N = g(c_0) \tag{5.1}$$

where $g(\cdot) \equiv f(I(\cdot))$.

The NIP makes its network infrastructure available to different SPs that charge customers for their services. In return, it gets a part $h < 1$ of the monthly fee as a commission for the infrastructure use. Hence, if N_i are the customers of SP_i, and p_i the fee it charges, the net profit of the NIP is

$$u_0(c_0, \mathbf{p}) = h \cdot \sum_{i=1}^{M} N_i \cdot p_i - d \cdot c_0 \tag{5.2}$$

where $\mathbf{p} = (p_1, p_2, .., p_M)$ and $d < 1$ is a constant amortizing the investment cost c_0 over a given time interval[2].

Furthermore, the NIP has peering agreements with one or more transit ISPs, which interconnect its network with the Internet. It undertakes the cost of leased line(s) and recovers it from the SPs using its network infrastructure (see cost-sharing mechanism in 5.2.1.5).

5.2.1.2. Service providers

Let $\mathcal{M}$ be the set of service providers, with $M = |\mathcal{M}|$. They offer services to the end users and maintain customer relationships with them. In this work, we consider M SPs providing the same service, Internet access connectivity.

SP_i charges a monthly fee p_i for its services. The monthly subscription fees charged by the SPs determine both the overall number of customers they will attract as a whole and their individual customer shares N_i, as shown in 5.2.1.3. Hence, both the overall and the individual demands are elastic to the charged fees. The revenue of SP_i equals $N_i \cdot (1-h) \cdot p_i$, after accounting for the commission h of the NIP for the operation and maintenance of the shared network infrastructure.

On the other hand, SP_i undertakes a part c_i of the cost of the leased line(s) that connect the shared network infrastructure to the Internet, as described in section 5.2.1.5. Hence, the net profit of SP_i out of the shared network infrastructure is

$$u_i = (1-h) \cdot N_i \cdot p_i - c_i \quad \forall i \in \mathcal{M} \tag{5.3}$$

[2] Typically, this would be the period of network operation before serious infrastructure upgrades are needed; or the desired recuperation time of the investment.

5.2.1.3. Network users

Network users join one of the competing SPs for gaining Internet access. This is a typical choice setting. Each user may consider and/or prioritize different criteria in evaluating her alternatives, *i.e.*, the M SPs.

Modeling the choice behavior of each user would add to the model accuracy but also to its complexity. Instead, we directly model the shares of the customer base seized by the different SPs as functions of the weighed fees charged by them. Namely, the number N_i of customers of SP i is given by

$$N_i = r_i(N, w_1p_1, w_2p_2, .., w_ip_i, .., w_Mp_M) \equiv r_i(N, \mathbf{w}, \mathbf{p}) \tag{5.4}$$

where r_i is a monotonically decreasing function of the charged fee p_i and the vector of weights $\mathbf{w} = (w_1, w_2, .., w_M)$ essentially summarize how SPs score beyond the fee criterion. For instance, the fee charged by a less reliable SP with slow response to customer requests would be given a higher weight, which make this SP look more "expensive" than it actually is. An example of such a function is the normalized exponential (softmax) function

$$r_i(N, \mathbf{w}, \mathbf{p}) \doteq \frac{N \cdot e^{-\delta \sum\limits_{j \in \mathcal{M}} p_j / M}}{1 + \sum_{j \in \mathcal{M} \setminus i} e^{w_ip_i - w_jp_j}} \tag{5.5}$$

The exponential in the numerator of (5.5) equals the portion of the potential customers N that subscribe to the Internet access service overall through any of the M SPs (effective market size). The constant δ modulates how aggressively end users respond to the price signal, here the average value of the subscription fee. The inverse of the denominator expresses the effective market share of SP_i (a number in [0,1]).

5.2.1.4. The community network dynamics

Community networks are crowd-sourced network infrastructures. They are typically initiated by small groups of people, who invest resources (equipment, time and effort) to build the first island of network connectivity. Then end users join the network, contributing their own equipment. This way, the network coverage can grow over time much larger than the original investment allowed. However, in many cases a CN follows a path towards extinction as the interest of users in it wanes over time.

These two possibilities for the CN evolution over time are captured by the simple model in [2]. Therein, the community members repeatedly iterate on joining the CN, if they have not done so, or staying with it, if they have already joined earlier, by considering its time-varying coverage Q(t), $0 \leq Q(t) \leq 1$, and a price signal P, which is a function of the fees charged for Internet access. For a single SP, this would be the fee itself; for more SPs, it could be the average of the fees they charge. Namely, a user u joins the CN at time t, gets service from one of the M SPs, and adds her own node to it, this way expanding the CN coverage, both geographically and in terms of actual users, as far as the net value

$$v_u = a_u \cdot Q(t) - P \tag{5.6}$$

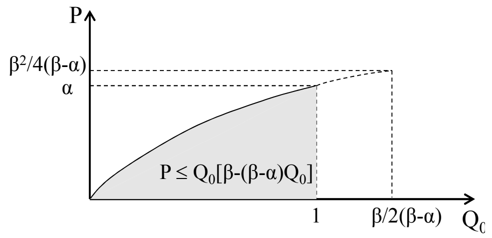
she extracts from it is non-negative. The user-specific factor a_u differentiates users with respect to how much they appreciate a given CN coverage. Higher a_u values imply readiness to pay higher fees for joining the CN.

These a_u factors follow a uniform distribution $\mathcal{U}[\alpha, \beta]$. Depending on the values of α and β, the price signal P and the initial CN coverage Q_0, the network evolves towards a different steady state coverage, Q_e. More specifically:

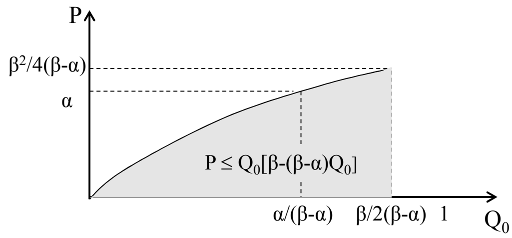
- when $\beta \leq 2 \cdot \alpha$, $Q_e = 0$ if $P > \alpha$, irrespective of the initial coverage Q_0, while $Q_e = 1$ for

$$P \leq Q_0 \cdot (\beta - (\beta - \alpha) \cdot Q_0) \tag{5.7}$$

Hence, the CN expands to full coverage as far as P remains below an increasing function of the initial



a. $\beta \leq 2\alpha$



b. $\beta > 2\alpha$

Figure 5.4: Feasible values of P *vs.* initial coverage Q_0 in a CN.

coverage (5.7); otherwise, it dies out. The higher the initial CN coverage, hence the initial investment in network infrastructure, the higher the fees that can be charged by the SPs, without inhibiting the trajectory of CN's coverage towards $Q_e = 1$.

- when $\beta > 2 \cdot \alpha$, there is more flexibility in the $Q_0 - P$ trade-off in (5.7). Now, $Q_e = 1$ as far as $P \leq \alpha$, but the CN can also reach a steady-state coverage

$$Q_e = Q_s = \frac{\beta + \sqrt{\beta^2 - 4 \cdot (\beta - \alpha) \cdot P}}{2 \cdot (\beta - \alpha)} < 1 \tag{5.8}$$

when higher fees

$$\alpha < P \leq \frac{\beta^2}{4(\beta - \alpha)} \tag{5.9}$$

are charged.

The two cases are shown in Fig. 5.4. When the network infrastructure is community-based, we refer to the NIP entity as *CNIP (Community Network Infrastructure Provider)*.

5.2.1.5. Cost sharing mechanism

One of the responsibilities of the NIP is to buy Internet transit connectivity from one or more transit ISPs and implement a cost sharing mechanism for distributing the Internet transit costs. This implies that the NIP measures how much transit traffic[3] is generated by the customers of each SP and computes a cost share for it accordingly.

Formally, let $C(q)$ be the cost function for total Internet transit traffic q produced by the customers of all M SPs, with $C(0) = 0$. If q_i is the traffic share produced by the customers of SP_i and $\overline{q} = (q_1, q_2, .., q_M)$ the overall network traffic profile, then a cost sharing rule ϕ associates to each $(C, M, \overline{q})$-tuple a vector $(c_1^M, c_2^M, .., c_M^M)$. Denoting the cost share of SP_i for traffic profile $\overline{q}$ and cost function $C(\cdot)$ with $c_i^M(C; \overline{q})$, it holds that:

$$c_i^M(C; \overline{q}) \geq 0 \; and \; \sum_{j=1}^{M} c_i^M(C; \overline{q}) = C(\sum_{j=1}^{M} q_i) \tag{5.10}$$

The first equation implies that the cost share is a non-negative number and becomes zero under $\overline{q} = \mathbf{0}$. The second equation ensures that the cost sharing rule is budget balanced: the cost shares of the M SPs make up

[3]It is expected that NIPs will operate Internet Exchange Points (IXPs) for serving traffic between the SPs that use its infrastructure so that this traffic does not contribute to the cost. Nothing changes from modeling point of view if this traffic is also accounted for in the operational cost.

exactly for the overall cost due to the total transit traffic $\sum_{j=1}^{M} q_i$.

Hereafter, we consider two such cost sharing rules; in particular, the average cost pricing [15] and the serial cost sharing rules [16].

Average cost pricing

Under average cost pricing (ACP), the cost share of SP_i is given by

$$c_i^M(C;\overline{q}) = \frac{q_i}{\sum_{j=1}^{M} q_j} C(\sum_{j=1}^{M} q_j) \tag{5.11}$$

Hence, the cost shares of SPs stand in proportion to the traffic their customers generate. An advantage of the Average Cost Pricings (ACPs) mechanism is that it is resilient to manipulations of the merge-split type: the total charge for an SP remains the same, even if it finds ways to split its traffic into smaller parts, *e.g.*, by spinning off more virtual entities, or merge its traffic with another SP, *e.g.*, by setting up a joint entity.

Serial cost sharing

Under serial cost sharing (SCS), the cost shares are computed iteratively after the network traffic shares are indexed in order of increasing volume, *i.e.*, $q_i \leq q_j, \forall i, j \in [1..M]$. Then the cost share of SP_1 equals

$$c_1^M(C;\overline{q}) = C(M \cdot q_1)/M; \tag{5.12}$$

and the cost shares of $SP_j,\ j > 1$ are given by (5.13).

$$c_j^M(C;\overline{q}) = c_{j-1}^M(C;\overline{q}) + \frac{C(\sum_{i-1}^{j-1} q_i + (M-j+1)\cdot q_j) - C(\sum_{i=1}^{j-2} q_i + (M-j+2)\cdot q_{j-1})}{M-j+1} \tag{5.13}$$

The main difference of the Serial Cost Sharings (SCSs) rule, when compared to the ACP one, lies in the way it divides the externalities the SPs generate on each other. When these externalities are negative, *i.e.*, the cost function $C(q)$ is convex, SPs generating little(much) traffic are charged by the SCS rule less(more) than what the proportional sharing of the ACP rule dictates. The opposite holds when externalities are positive, *i.e.*, when $C(q)$ is a concave function. Then, SPs generating much(little) traffic are charged lower(higher) than what proportionality dictates.

The two cost sharing mechanisms are compared axiomatically in [15]. In this work, we consider them in the context of the infrastructure sharing game we describe in section 5.2.2. In particular, We are interested in the equilibria they induce for the strategies of the NIP and SP entities and the way the crowdsourced dynamics of CNs can catalyze synergies and foster their sustainability.

5.2.2. The network infrastructure sharing game

In Section 5.2.1, we analyzed the different actors and their stakes in this layered network model, where the network infrastructure is owned and operated by one NIP entity (or owned by many and operated by one CNIP entity, when a community network is involved), and shared by many SPs providing services over it to end users.

The possible profits of the NIP and the SPs out of this layered network model depend on the original investment of the NIP on network infrastructure, the pricing strategies of the SPs, and the cost sharing mechanism that determines how the operational costs of the infrastructure are shared among the SPs. SPs need to compete against each other for attracting end users as customers but should also coordinate with the NIP in generating a market large enough to render this business model profitable for all of them.

We capture the interactions of the actors within the framework of leader-follower games. The leader player in our case is the NIP that invests an amount x_0 in network architecture. This investment determines the coverage of the network, hence the *potential customer base*, through (5.1). When the NIP is a CNIP, the investment determines the initial network coverage Q_0 but this may grow or shrink to a different value Q_e, as discussed in 5.2.1.4.

The followers are the SPs. The service subscription fees determine the *actual customer base* and the customer shares the different SPs attract. At the same time, they define the net profit of the (C)NIP (see 5.2), and through the cost sharing mechanism, the net profit of the SPs (see 5.3).

5.2.2.1. The SP pricing game

For a given choice of the c_0 value by NIP, the choice of service subscription fees by SPs gives rise to the strategic-form game $G_M(c_0) = \langle \mathcal{M}, (p_i)_{i\in\mathcal{M}}, (u_i)_{i\in\mathcal{M}} \rangle$ where:

- $\mathcal{M}$ is the set of player-SPs;
- $(p_i)_{i\in\mathcal{M}}$ are the (symmetric) sets of strategies of the M SPs; and
- u_i is the payoff function for SP_i, as given by (5.3).

$G_M(c_0)$ is a continuous game; Nash Equilibria (NE) strategies can be found at the intersection of the best response functions of the M SPs. To find them, we maximize the payoff function u_i of each SP_i with respect to its own strategy p_i. Therefore, by first-order optimality conditions:

$$\frac{\partial u_i}{\partial p_i} = (1-h)\frac{\partial N_i(x_0, p_i, p_{-i})p_i}{\partial p_i} - \frac{\partial c_i^M(c_0, p_i, p_{-i})}{\partial p_i} = 0 \tag{5.14}$$

where the dependence of both customer shares N_i and cost shares c_i on the fees and the NIP investment are explicated, and p_{-i} denotes the set of fees charged by all SPs except for SP_i.

5.2.2.2. The determination of the initial investment by the NIP

If $\overline{p}(c_0)$ are the fee values that result from the system of equations (5.14), the NIP will seek to maximize its net profit by solving the following optimization problem

$$\begin{aligned} \max_{c_0,\mathbf{p}} \;\; & u_0(c_0, \mathbf{p}) \\ s.t. \;\; & (5.14) \\ & c_0 \geq 0, \; p_i > 0, \; i \in [1..M] \end{aligned}$$

where the net profit of NIP is given by (5.2).

5.2.2.3. Example: network infrastructure sharing by 2 SPs

We consider as case study the sharing of network infrastructure by 2 SPs under the average cost pricing scheme and a concave cost function $C(q) = c \cdot log(q)$. Assuming a concave cost function matches the current pricing of wholesale bandwidth which increases sub-linearly with the purchased bandwidth. We distinguish between a commercial (for-profit) NIP entity and a community-based one.

Commercial NIP

The share of network users captured by SP_1 and SP_2 are:

$$N_1 = \frac{N \cdot e^{-\delta \frac{p_1+p_2}{2}}}{1 + e^{w_1 p_1 - w_2 p_2}}, \; N_2 = \frac{N \cdot e^{-\delta \frac{p_1+p_2}{2}}}{1 + e^{w_2 p_2 - w_1 p_1}} \tag{5.15}$$

where N is a function of the investment by the NIP, see (5.1).
If q_{av} is the average user traffic per month, the transit connectivity cost shares of the two SPs become (5.11)

$$c_1^2 = \frac{C(Ne^{-\delta \frac{p_1+p_2}{2}} \cdot q_{av})}{1 + e^{w_1 p_1 - w_2 p_2}} \; c_2^2 = \frac{C(Ne^{-\delta \frac{p_1+p_2}{2}} \cdot q_{av})}{1 + e^{w_2 p_2 - w_1 p_1}} \tag{5.16}$$

Combining (5.3) with (5.15) and (5.16), the payoff functions of the two SPs can be written:

$$u_1 = \frac{(1-h)Ne^{-\delta \frac{p_1+p_2}{2}} p_1 - clog(Nq_{av}) + \frac{c\delta(p_1+p_2)}{2}}{1 + e^{w_1 p_1 - w_2 p_2}}$$
$$u_2 = \frac{(1-h)Ne^{-\delta \frac{p_1+p_2}{2}} p_2 - clog(Nq_{av}) + \frac{c\delta(p_1+p_2)}{2}}{1 + e^{w_2 p_2 - w_1 p_1}}$$

Applying the first-order optimality conditions in (5.14), and defining

$$\gamma = e^{w_1 p_1 - w_2 p_2}, \; y = e^{-\delta \frac{p_1+p_2}{2}} \tag{5.17}$$
$$\theta(y, p_i) = N(1-h)yp_i - clog(Nq_{av}) + \frac{c\delta(p_1+p_2)}{2}$$
$$\xi(y, p_i) = N(1-h)y(1 - \frac{\delta p_i}{2}) + \frac{c\delta}{2}$$

we get the two equations that NE values of p_1 and p_2 must be satisfying:

$$\begin{aligned} w_1 \cdot \gamma \cdot \theta(y, p_1) - (1+\gamma) \cdot \xi(y, p_1) = 0 \\ w_2 \cdot \theta(y, p_2) - (1+\gamma)\xi(y, p_2) = 0 \end{aligned} \tag{5.18}$$

In turn, the NIP entity is faced with the optimization problem

$$\begin{aligned} \max_{x_0, p_1, p_2} \quad & h \cdot y \cdot g(x_0)(\frac{p_1}{1+\gamma} + \frac{p_2 \cdot \gamma}{\gamma + 1}) - x_0 \\ s.t. \quad & (5.17), (5.18), \\ & x_0 \geq 0, \; p_i > 0, \; i \in [1..M] \end{aligned} \tag{5.19}$$

This is a non-linear continuous optimization problem that can be solved with numerical techniques.

Community-based NIP

The difference when the network infrastructure is community-based is that the coverage of the network may grow thanks to potential contributions of resources by the community. The intensity and the impact of these contributions depend on the original investment c_0 and the fees charged by the two SPs, as described in section 5.2.1.4. From a problem formulation point of view, The equations (5.15)-(5.16) still hold, if we replace the term $y = Ne^{-\delta\frac{p_1+p_2}{2}}$ with $N \cdot Q_e$, as described in section 5.2.1.4.

Note that, in the general case, Q_e is no longer a continuous function of (c_0, p_1, p_2) but rather takes discrete values in $\{0, Q_s, 1\}$. However, depending on the spread of the a_u values distribution, we can set $Q_e = 1$ under the additional constraints (5.7) and $P \leq \alpha$ (when $\beta \leq 2\alpha$); or replace it by (5.8) under the additional constraints (5.7) and $P \leq \frac{\beta^2}{4(\beta-\alpha)}$ (when $\beta > 2\alpha$).

5.2.3. Main findings and implications

The model evaluation is ongoing work; in the remainder of the section, we present preliminary results outlining main trends.

5.2.3.1. Benefits of grassroots community network infrastructure providers

Figure 5.5 compares the two possibilities to come up with network infrastructure described in section 5.2.2 with respect to the achievable revenues for the different actors (NIP and SPs) at the equilibrium states the cost sharing mechanisms induce.

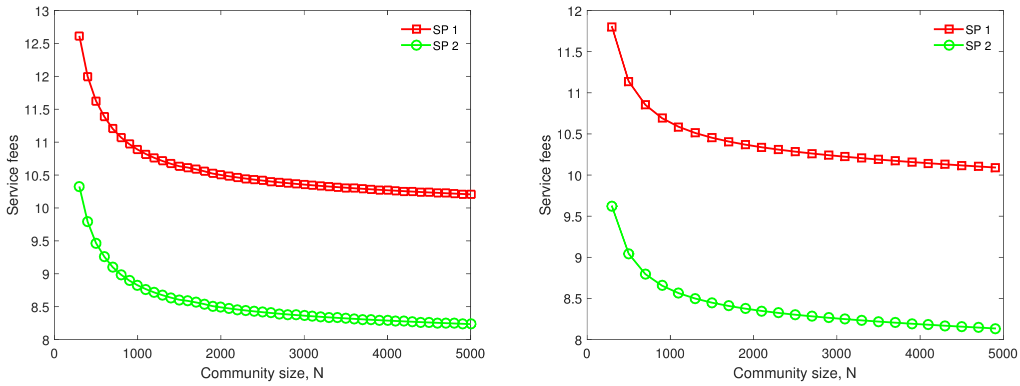
More specifically, when the investment cost for deploying the network infrastructure is fully on the NP entity (left column), the investment that has to be made by it is in the order of three times higher than the one a CN NP has to make (Fig. 5.5(b)). In the second case, part of the infrastructure costs can be undertaken by community members that progressively join the CN, after the initial seed investment is made by the CNIP. These savings result in analogously higher net profit for the CNIP, as shown in 5.5(d). Namely, the CNIP enjoys practically the same revenue, due to its commission from the revenues of SPs, at much smaller initial investment.

On the other hand, the fees that SPs charge at the game equilibria are comparable in the two cases (slightly higher for smaller community sizes in the generic NIP case but they tend to converge for larger sizes). Hence, the attracted number of subscribers and the net profits of the SPs are almost identical in the two scenarios, as can be seen in Figs. 5.5(c) and 5.5(d), respectively.

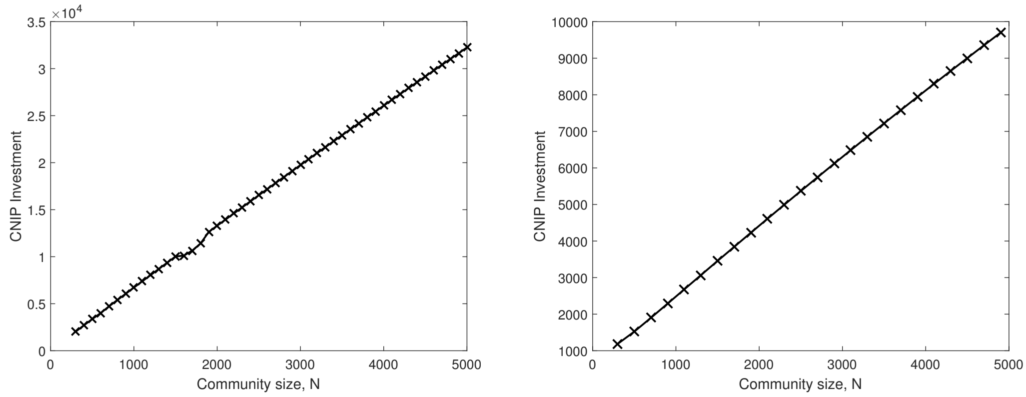
5.2.3.2. The (non-)impact of cost-sharing mechanisms

Figure 5.6 focuses on the scenario, where the shared network infrastructure is a CN and compares the two cost sharing rules described in section 5.2.1.5.

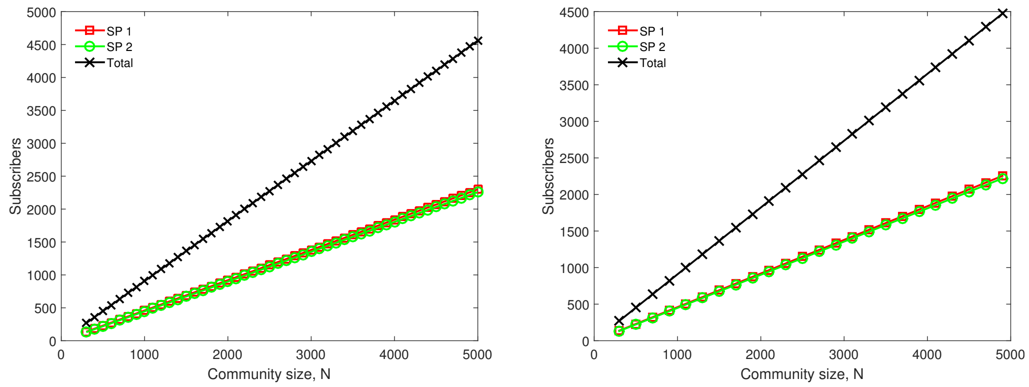
Notably, the actual cost sharing mechanism in place appears to have negligible impact on what each entity can gain out of the synergy. The equilibrium strategies the two alternatives for sharing the costs induce, result in almost identical revenues for the SPs and the CNIP in the two cases. The smaller initial investment by the CNIP at the equilibrium under serial cost sharing is balanced by the slightly lower fees at the equilibrium. As a result the subscription revenues for the two SPs are slightly higher under the average cost pricing rule. On the other hand, the smaller investment cost for the CNIP under serial cost sharing is outweighed by the smaller commission out of the SP subscription revenues such that its net profit under the two alternatives is practically identical.



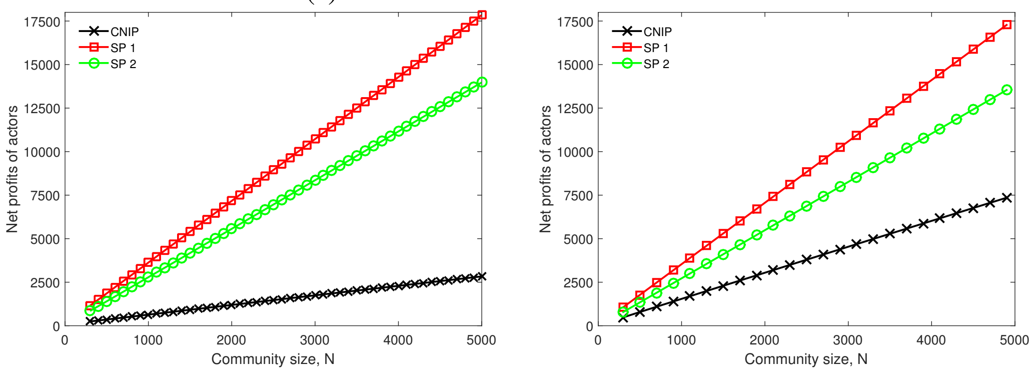
(a) Fees charged by the 2 SPs



(b) Investment by the NP



(c) Number of attracted subscribers



(d) Actor revenues

Figure 5.5: Charged fees and attracted subscribers by the two SPs, investment from the NP, and resulting revenues for all three entities at the equilibrium when the infrastructure is a CN (right) and when it is not (left).

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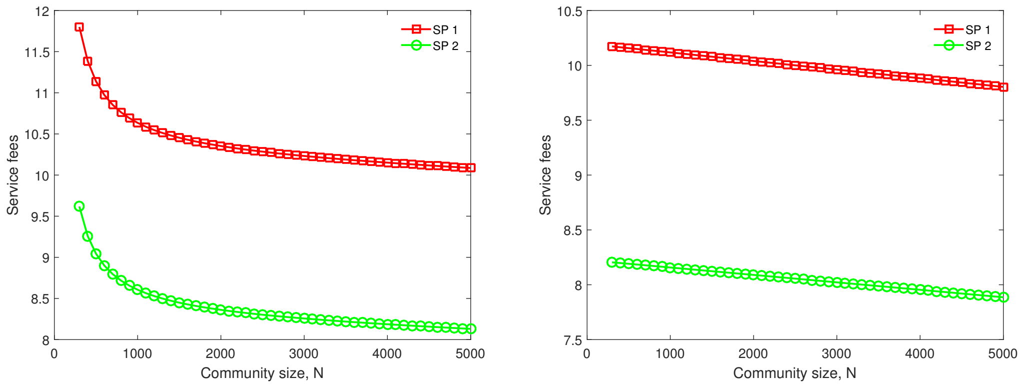
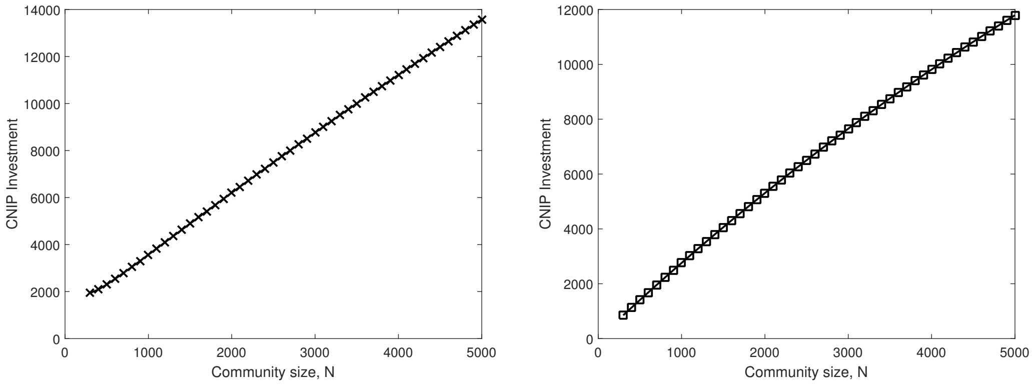
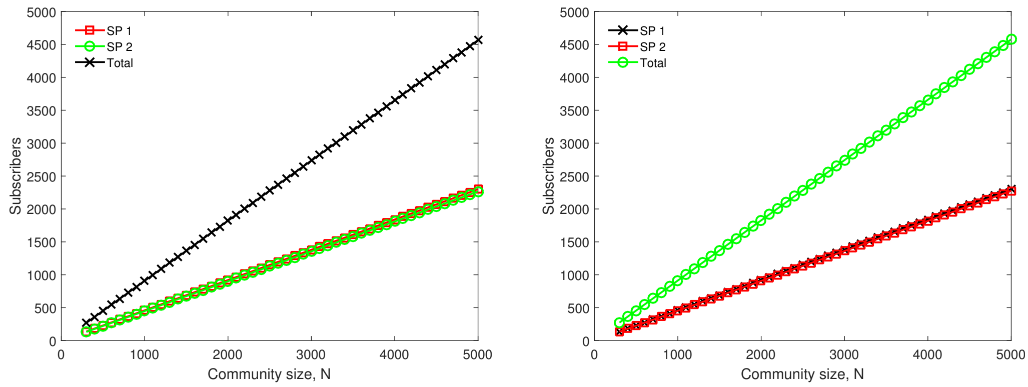


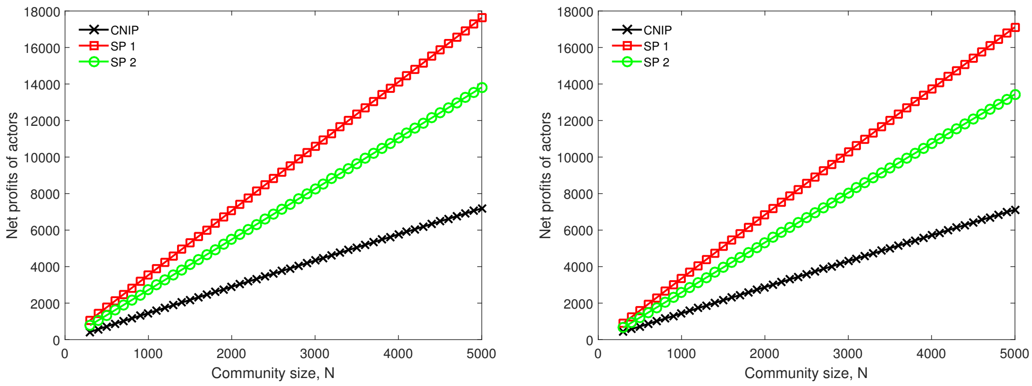
(a) Fees charged by the 2 SPs



(b) Initial investment by the CNIP



(c) Number of attracted subscribers



(d) Actor revenues

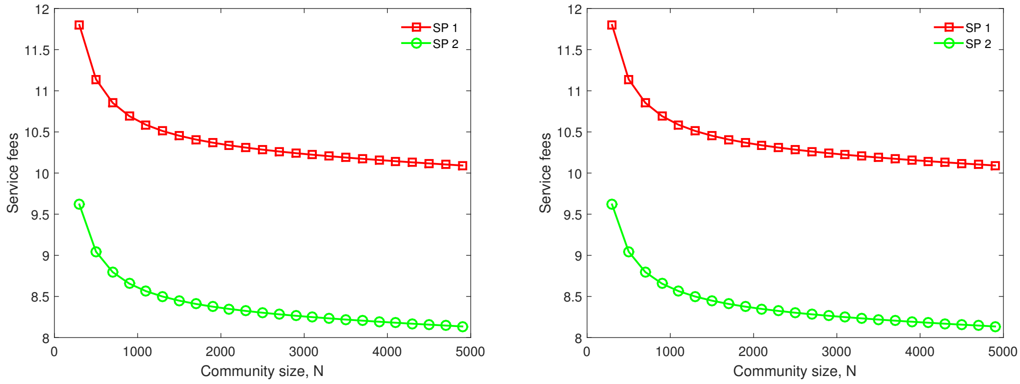
Figure 5.6: Charged fees and attracted subscribers by the two SPs, investment from the CNIP, and resulting revenues for all three entities at the equilibrium under the Average Cost Proportional (right) and the Serial Cost Sharing (left) rules.

5.2.3.3. Sensitivity to the wealthiness of the community

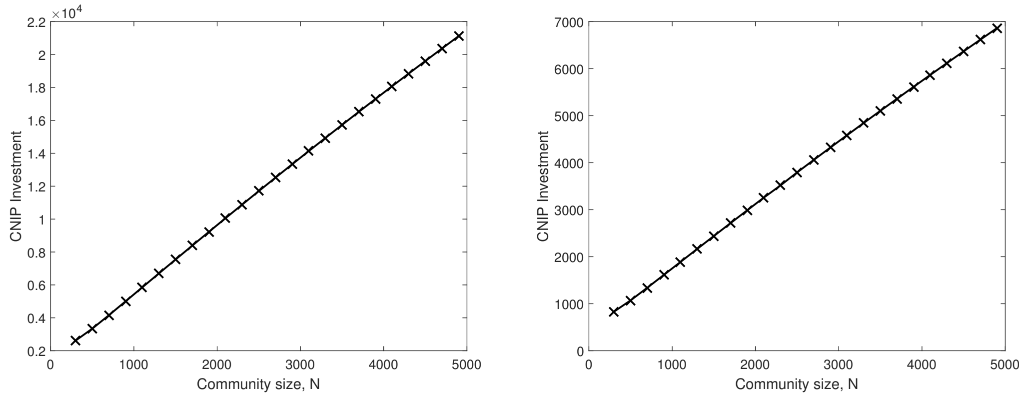
When the community members are willing to pay more for Internet connectivity, the actor with the highest profit margins is the CNIP (Fig. 5.7). The required initial investment to achieve full coverage at steady state (see (5.7)) for given subscription fees is significantly smaller. Hence, the CNIP can save more money and use it for infrastructure maintenance purposes.

Neither the equilibrium subscription fees nor the attracted subscribers appear to be affected by what community members are willing to pay, at least under the average cost pricing rule used in these experiments.

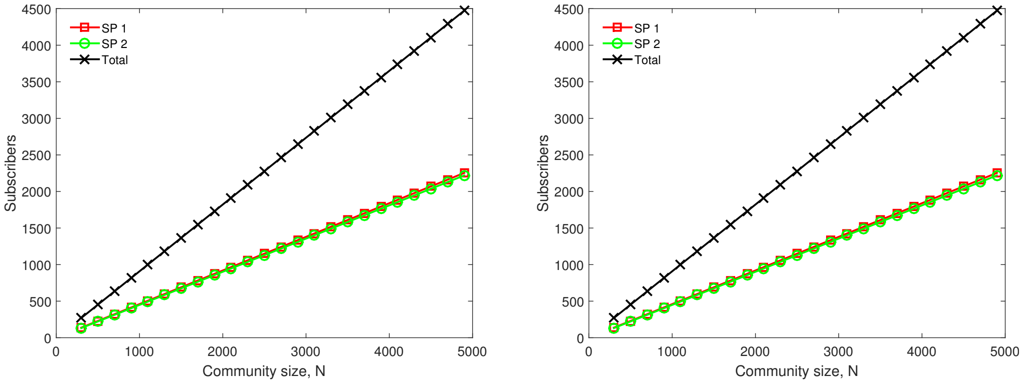
> **Key takeaway for CNs:** These, preliminary, results suggest (and at the same time confirm evidence coming from guifi.net) that synergies with for-profit entities can be sustainable. Indeed, the way a CNIP grows, by leveraging the community dimension and the social links within the community, turns otherwise non-profitable market scenarios into viable synergies with returns that can ensure their sustainability.
> That said, we have to recall that the management of a CPR requires participation and engagement from the stakeholders, which introduces a layer of complication compared to the well established customer/vendor model in typical market scenarios.



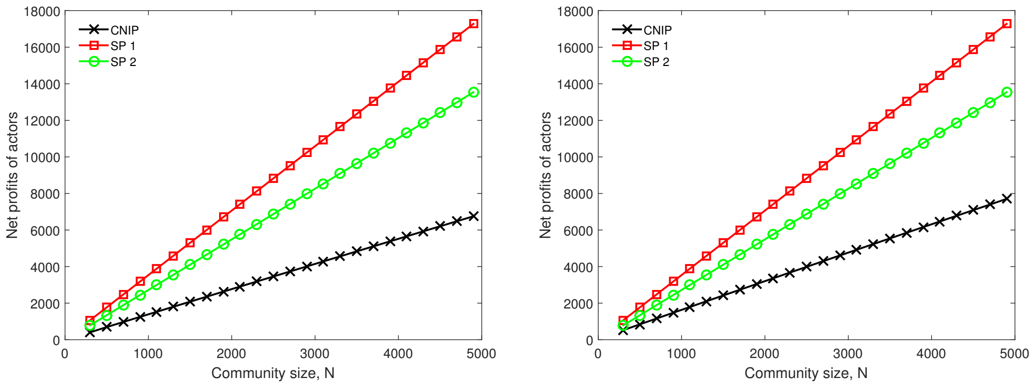
(a) Fees charged by the 2 SPs



(b) Initial investment by the CNIP



(c) Number of attracted subscribers



(d) Actor revenues

Figure 5.7: Charged fees and attracted subscribers by the two SPs, investment from the CNIP, and resulting revenues for all three entities at the equilibrium under the Average Cost Proportional sharing rule, when $\alpha = 12, \beta = 16$ (left) and when $\alpha = 15, \beta = 22$ (right)

5.3. Community clouds

5.3.1. System model

The three types of actors in the model are the service providers, the cloud infrastructure operator and the end users. The service providers are mainly interested in achievable revenues, whereas the cloud infrastructure operator aims at ensuring the sustainability of the infrastructure, both in financial and social inclusion terms.

Note that the concept of Community Cloud is a community-based specific instance of a computing model that is now attracting a lot of attention from the academia and the market, the so-called "Edge Computing" model [17]. The term Edge Computing describes the trend of moving computational power from cloud systems located in server farms (or Content Delivery Networks) to the edges of the network, close to the users. This is a key element of the 5G networking model, and the study of community clouds we performed in netCommons (see also the deliverables of WP3, like [18]) is an effort to upgrade the technological assets available for a CN in the light of the recent technological advances.

5.3.1.1. Business actors

Service providers

Service providers wish to build a shared infrastructure and deploy services to users within reach of the infrastructure in exchange for some compensation. They invest money in building this infrastructure and the return payments are related to the quality of service they can offer. The infrastructure is made of servers which can be used to deploy services in a cloud-based model, these servers are co-located with the nodes of the network and can be orchestrated to dynamically provide services to the users.

Formally, let $\mathcal{S}$ be the set of n budget-limited Service Providers (SPs), which are interested in investing money in a common computing infrastructure. These SPs will use this infrastructure later in a shared fashion to provide services to their customers. Each SP i has a unit budget and may invest a portion x_i of its budget as a contribution towards building the infrastructure. It can the use the remaining portion $(1 - x_i)$ of the budget for leasing the resulting common infrastructure and providing its services.

Infrastructure operator

An infrastructure operator (IO) regulates the interaction of different SPs and physically builds the infrastructure. It is responsible for: (*i*) collecting SP investments, (*ii*) building the infrastructure out of these investments, (*iii*) allocating resource slices to SPs for offering their services, (*iv*) charging SPs for the use of the infrastructure, and (*v*) sharing the revenue among SPs (Fig. 5.8).

In this work, we abstract the infrastructure as a set of resources, in particular, as one type of resource (*e.g.*, time shares in virtual machines). Let $\mathbf{x} = (x_1, \ldots, x_n)$ be an SP investment vector and $b(\mathbf{x})$ be a continuous function that translates the investment vector $\mathbf{x}$ into an amount of that resource. For example, for computational resources, $b(\mathbf{x}) = \frac{1}{c}\sum_{i=1}^{n} x_i$ denotes the amount of computational capacity available as a result of total investment $\sum_{i=1}^{n} x_i$, where c is the cost per unit of computational capacity. Function $b(\mathbf{x})$ is assumed to be known to the SPs; thus, SPs know the impact of their collective investments on expanding the infrastructure with resources. Hereafter, unless otherwise stated, we assume that $c = 1$, thus $b(\mathbf{x}) = \sum_{i=1}^{n} x_i$.

5.3.1.2. Slice allocation

After the infrastructure is built, the IO allocates requested resource slices to SPs for deploying and running their services. Without loss of generality, we assume that each SP i deploys one service and that all available

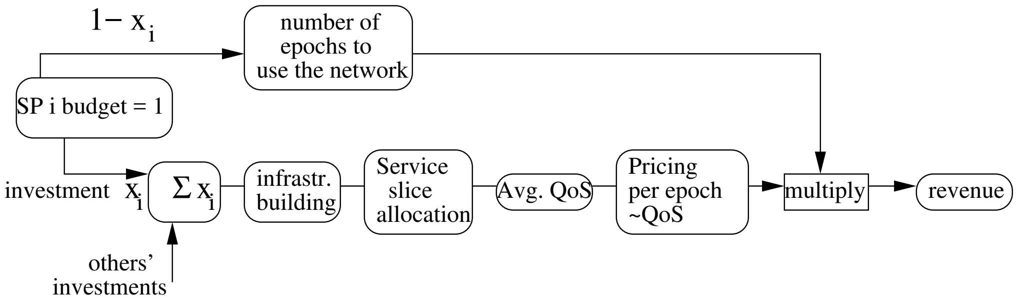


Figure 5.8: Each SP i invests a portion x_i of its budget towards building the infrastructure, and the rest, $1 - x_i$ for using the infrastructure in a shared fashion for service provision. A single SP is depicted. Infrastructure building, service slice allocation and pricing are performed by the IO.

slices have the same resource requirements. Hence, the total number of infrastructure slices created by SP investments is $\frac{b(\mathbf{x})}{C}$, where C is the amount of resource requirements per slice.

The IO employs a rule $\mathbf{s} = (s_1, \ldots, s_n)$ for allocating service slices to SPs, where s_i is the portion of the total number of slices assigned to SP i, such that $\sum_{i=1}^{n} s_i = 1$. Thus, SP i gets

$$k_i(\mathbf{x}) = \frac{s_i}{C} \sum_{i=1}^{n} x_i \,. \tag{5.20}$$

slices. For instance, slices may be split equally among SPs, *i.e.*, $s_i = \frac{1}{n}$ for all i; or according to a proportional allocation rule, where the number of allocated slices to each SP i is proportional to the demand for their service, *i.e.*, $s_i = d_i / \sum_{j=1}^{n} d_j$.

5.3.1.3. Service provision and revenue generation

Each SP i uses the portion $(1 - x_i)$ of its budget that was not invested in infrastructure to lease infrastructure resources for offering services to its customers. Our payment model is inspired from the realistic situation, where slices are leased for some amount of time, *e.g.*, in the order of months, against some fee f per unit of time, called an epoch. The number of epochs for which slices can be leased is $(1 - x_i)/f$. We assume a unit fee $f = 1$ per epoch for each SP so that SP i leases slices for $(1 - x_i)$ epochs.

The anticipated user demand per epoch for the service of SP i is d_i. SP i can provide its service with a given quality. We assume that the average level of quality of service provisioned by SP i is uniform across users according to a continuous function $q_i(\cdot)$ which depends on d_i, and on the amount of allocated slices $k_i(\mathbf{x})$, and thus on the investment vector $\mathbf{x}$. This quality of service level translates to money flow for the SP through a service charging model per epoch and per unit of demand, comprising a pricing function $p_i(q_i(\cdot))$. Finally, the total revenue $u_i(\cdot)$ comes out of total demand over all epochs, and

$$u_i(\mathbf{x}) = (1 - x_i) d_i p_i(q_i(\mathbf{x})) \tag{5.21}$$

Proposition 3. *Function $u_i(\cdot)$ is a concave function of x_i if* (i) *$q_i(\cdot)$ is decreasing and convex function of x_i, and $p(\cdot)$ is decreasing, and convex or linear function of $q_i(\cdot)$, or* (ii) *$q_i(\cdot)$ is increasing and concave function of x_i, and $p(\cdot)$ is increasing, and concave or linear function of $q_i(\cdot)$.*

The proof can be found in Appendix III.

Example

Let $k_i(\mathbf{x})$ be the total computational power of SP i (in cycles per second) as given by (5.20). Assuming a stream of service requests for SP i arising according to a Poisson distribution with rate d_i requests per unit of time, a plausible quality of service metric is the *average delay per request* coming out of an M/M/1 service queue assumption; this is given by

$$q_i(\mathbf{x}) = \frac{1}{k_i(\mathbf{x}) - d_i}\,. \tag{5.22}$$

A pricing function $p_i(\cdot)$ charges customers according to average request delay; this can be a convex decreasing function of q_i, *e.g.*, a piecewise-linear convex function consisting of line segments of different slopes denoting the rates of decrease of price per unit of delay increase. In its simplest form, this piece-wise linear convex function has two segments,

$$p_i(q_i(\mathbf{x})) = \begin{cases} a_i - D_i q_i(\mathbf{x}), & \text{if } q_i(\mathbf{x}) \le a_i/D_i \\ 0, & \text{otherwise,} \end{cases} \tag{5.23}$$

where a_i is the price for zero delay and a_i/D_i is the maximum delay for which there is nonzero charge. If the epoch duration is τ, the derived average utility per epoch for all demand is $d_i \tau p_i(q_i(\mathbf{x}))$ and the total revenue is

$$u_i(\mathbf{x}) = (1 - x_i) d_i \tau p_i(q_i(\mathbf{x}))\,. \tag{5.24}$$

5.3.1.4. Revenue sharing

The IO may distribute the total incurred revenue among SPs, after possibly withholding a certain percentage as commission. Revenue allocation needs to be done so that investments are encouraged. Shapley value [19] arises as an appropriate mechanism to allocate the worth of a coalition among its participants, where the worth of a coalition is the total maximum revenue obtained by its members through cooperation. In our case the worth of coalition $\mathcal{S}$ of SPs is

$$v(\mathcal{S}) = \max_{\mathbf{x} \ge \mathbf{0}} \sum_{i \in \mathcal{S}} u_i(\mathbf{x}), \quad \text{such that } u_i(\mathbf{x}) \ge 1 \text{ for all SPs } i \tag{5.25}$$

The constraint plays the role of a participation incentive for each SP, in the sense that the resulting revenue for each SP after investing and using the network should be at least as much as its initial budget.

Among the class of revenue distribution mechanisms, the Shapley-value one is the only one that satisfies desirable properties such as fairness, efficiency, symmetry and strong monotonicity [20], [21]. The Shapley value for SP i is

$$\phi_i^v(\mathcal{S}) = \frac{1}{n!} \sum_{\pi \in \Pi} \Delta_i(v, \mathcal{S}(\pi, i))\,, \tag{5.26}$$

where Π is the set of all $n!$ orderings of $\mathcal{S}$, $\mathcal{S}(\pi, i)$ is the set of SPs preceding i in ordering π, and

$$\Delta_i(v, \mathcal{A}) = v(\mathcal{A} \cup \{i\}) - v(\mathcal{A}) \tag{5.27}$$

is the marginal contribution of SP i to subset $\mathcal{A}$. The Shapley value is interpreted as the expected marginal contribution of SP i to various subsets of SPs that precede i in a uniformly distributed random ordering of $\mathcal{S}$. Due to efficiency of the mechanism, it is

$$\sum_{i=1}^{n} \phi_i^v(\mathcal{S}) = v(\mathcal{S})\,. \tag{5.28}$$

Alternatively, the Shapley value can be written as

$$\phi_i^v(\mathcal{S}) = \frac{|\mathcal{A}|!(n-|\mathcal{A}|-1)!}{n!} \sum_{\mathcal{A}\subseteq\mathcal{S}\setminus\{i\}} \left(v(\mathcal{A}\cup\{i\}) - v(\mathcal{A})\right), \tag{5.29}$$

which is the average marginal contribution of SP i over all possible permutations in which coalition $\mathcal{S}$ can be formed. For $n=2$ SPs, the Shapley value-shares are

$$\phi_1^v(\mathcal{S}) = \frac{1}{2}[v(\{1\}) + v(\{1,2\}) - v(\{2\}] \tag{5.30}$$

and

$$\phi_2^v(\mathcal{S}) = \frac{1}{2}[v(\{2\}) + v(\{1,2\}) - v(\{1\}] \tag{5.31}$$

In the next section, we drop "v" from the notation of Shapley value so as to simplify notation.

5.3.2. The SP investment game

SPs are strategic in the portion of budget they invest towards building the common infrastructure. On the one hand, an SP would like to conserve its budget and invest as little as possible to the common infrastructure. Then, it would be able to reap the benefits of the shared infrastructure out of service provisioning to its customers for a larger number of epochs. On the other hand, the larger the invested amount the more resource slices are created and hence the SP share of resource slices is going to be larger as well. In turn, a larger number of resource slices implies better quality of service and thus higher revenue per epoch from provided services. Overall, the tradeoff in deciding how much to invest amounts to deciding between *providing service for more epochs but possibly at lower quality, and thus with fewer earnings per epoch, versus providing better quality services and thus more earnings per epoch but for smaller number of epochs.*

A key quantity for all involved entities is the total revenue of all SPs. It reflects the total earnings of SPs and the utility of end-users that pay SPs to enjoy services. Furthermore, the IO benefits from high total SP revenue since this implies a higher commission as percentage of the revenue and guarantees the sustainable operation of the infrastructure.

If SPs jointly coordinate their strategies, they seek the investment policy $\mathbf{x}$ that maximizes total revenue, *i.e.*, they would jointly solve

$$\max_{\mathbf{x}\geq\mathbf{0}} \sum_{i=1}^{n} u_i(\mathbf{x}), \quad \text{such that} \ \ u_i(\mathbf{x}) \geq 1 \ \text{for all SPs} \ i\,. \tag{5.32}$$

We denote this global optimal solution by $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$.

5.3.2.1. Investment game under revenue sharing

Each SP is interested in determining the invested amount that maximizes its own revenue. We assume that the resource slice allocation rule $\mathbf{s}$ is fixed and that the IO applies a total revenue sharing rule which is announced a priori to SPs. We are after the revenue sharing rule $\boldsymbol{\phi} = (\phi_1, \ldots \phi_n)$ among the n SPs, such that if SPs are selfish and strategic in the sense above, their selfish behavior results in the global optimal investment solution. In other words, if each SP i aims at maximizing its share ϕ_i obtained from the sharing rule $\boldsymbol{\phi}$, the collective selfish investment strategy of SPs should maximize total revenue as well.

In order to stress the dependence of the revenue share of each SP i on the investment strategy $\mathbf{x}$, we also write $\boldsymbol{\phi}_i(\mathcal{S}, \mathbf{x})$. For the same reason, we write the worth of coalition $\mathcal{S}$ as $v(\mathcal{S}, \mathbf{x})$. We write $\mathbf{x}_{-i}$ to denote the investment policy of all SPs except i. Given a revenue sharing rule $\boldsymbol{\psi}$, each SP solves the following optimization problem,

$$\max_{x_i \geq 0} \psi_i(\mathcal{S}, x_i, \mathbf{x}_{-i}), \quad \text{such that } u_i(\mathbf{x}) \geq 1\,. \tag{5.33}$$

A Nash equilibrium $\mathbf{x}^0(\boldsymbol{\psi}) = (x_1^0(\boldsymbol{\psi}), \ldots, x_n^0(\boldsymbol{\psi}))$ is an investment vector such that for each SP i, it is $\psi_i(\mathcal{S}, x_i^0(\boldsymbol{\psi}), \mathbf{x}_{-i}^0(\boldsymbol{\psi})) \geq \psi_i(\mathcal{S}, x_i, \mathbf{x}_{-i}^0(\boldsymbol{\psi}))$ for any other investment strategy $x_i \neq x_i^0(\boldsymbol{\psi})$, given that all other SPs do not change their strategies. In a Nash equilibrium, no SP has an incentive to unilaterally deviate from its chosen strategy.

Proposition 4. *If revenue sharing is performed based on the Shapley value of each SP i, i.e. if $\boldsymbol{\psi} = \boldsymbol{\phi}$, then if each SP i applies the optimal investment policy x_i^*, it maximizes its Shapley value share i.e., it is*

$$x_i^* = \arg\max_{x_i \geq 0} \phi_i(\mathcal{S}, x_i, \mathbf{x}_{-i})\,. \tag{5.34}$$

Thus, by adopting the optimal investment strategy x_i^*, an SP i optimizes its own revenue share, if sharing is performed according to its Shapley value. Further, we show the following:

Proposition 5. *Under the Shapley value revenue sharing mechanism, each optimal investment strategy $\mathbf{x}^*$ is a Nash equilibrium, i.e. it is $\mathbf{x}^* = \mathbf{x}^0(\boldsymbol{\phi})$.*

Proof. Assume that the optimal investment strategy $\mathbf{x}^*$ is not a Nash equilibrium. Then there would exist an SP i that could change its strategy from x_i^* to x_i and achieve a share $\phi_i(\mathcal{S}, x_i, \mathbf{x}_{-i}^*) > \phi_i(\mathcal{S}, x_i^*, \mathbf{x}_{-i}^*)$. This is a contradiction since x_i^* is an optimal investment strategy for SP i. $\square$

Therefore, under the Shapley value revenue-sharing rule, the selfish revenue-share maximization strategy of each SP coincides with the global optimal investment strategy that maximizes total incurred revenue. This scenario is desirable for all stakeholders of the shared infrastructure, the IO, the SPs and the end-users.

5.3.2.2. Investment game under resource slicing

Now we assume that revenue sharing cannot be applied either because SPs may be unwilling to abide to the IO rules, or because the IO cannot enforce such a sharing rule. In that case, each SP will select its investment strategy so as to maximize its own revenue through solving

$$\max_{x_i \geq 0} u_i(\mathbf{x}), \quad \text{such that } u_i(\mathbf{x}) \geq 1\,. \tag{5.35}$$

A Nash equilibrium $\tilde{\mathbf{x}}$ is an investment vector such that for each SP i, it is $u_i(\tilde{x}_i, \tilde{\mathbf{x}}_{-i}) \geq u_i(x_i, \tilde{\mathbf{x}}_{-i})$ for any other investment strategy $x_i \neq \tilde{x}_i$.

The Nash equilibrium can be computed numerically as follows. We write the Lagrangian for problem (5.35) for each SP i. We apply the necessary and sufficient KKT conditions for each Lagrangian, and we solve the resulting system of equations. Since $u_i(\mathbf{x})$ is concave in x_i, the Nash equilibrium is unique.

The *Price of Anarchy (PoA)* for the game above is

$$\text{PoA} = \frac{\sum_{i=1}^n u_i(\mathbf{x}^*)}{\sum_{i=1}^n u_i(\tilde{\mathbf{x}})} \geq 1\,. \tag{5.36}$$

Example: Nash equilibrium for $n = 2$

Consider a fixed resource slice allocation rule given by $s_1 \in (0,1)$ and $s_2 = 1 - s_1$. The necessary and sufficient conditions for $\mathbf{x}^* = (x_1^*, x_2^*)$ to be a Nash equilibrium result in

$$\frac{\partial u_1(\mathbf{x}^*)}{\partial x_1} = 0, \quad \text{and} \quad \frac{\partial u_2(\mathbf{x}^*)}{\partial x_2} = 0\,, \tag{5.37}$$

which lead to the system of equations,

$$\begin{aligned} a_1 g_1^2(x_1,x_2) - D_1 C g_1(x_1,x_2) - (1-x_1)D_1 C = 0, \\ a_2 g_2^2(x_1,x_2) - D_2 C g_2(x_1,x_2) - (1-x_2)D_2 C = 0, \end{aligned} \tag{5.38}$$

where $g_1(x_1,x_2) = s_1(x_1+x_2) - Cd_1$, $g_2(x_1,x_2) = s_2(x_1+x_2) - Cd_2$. The solution gives the Nash equilibrium point.

Finding the optimal slice allocation rule

The IO wishes to alleviate the negative effect of selfish SP investment on total revenue by choosing an appropriate resource slice allocation rule $\mathbf{s}$. We write the Nash equilibrium point as $\tilde{\mathbf{x}}(\mathbf{s})$ to stress its dependence on $\mathbf{s}$. The IO wishes to minimize the PoA. If the numerator of PoA in (5.36) is fixed, then the IO precomputes the Nash equilibrium as function of $\mathbf{s}$ and then solves

$$\max_{\mathbf{s}} \sum_{i=1}^{n} u_i(\tilde{\mathbf{x}}(\mathbf{s})) \ \text{ such that } \ \sum_{i=1}^{n} s_i = 1\,. \tag{5.39}$$

5.3.3. Main findings and implications

Given the provisions and assumptions of our model about the strategic interactions of SPs when sharing a shared infrastructure like community clouds, which can be made up by their own investments, our main findings are that: (*i*) the selfish revenue-maximization investment strategy coincides with the global optimal one that maximizes the total revenue of the SPs, if the total revenue is shared among SPs according to the Shapley value mechanism, (*ii*) when revenue sharing is not possible, the negative impact of SP selfishness on total revenue can be alleviated as much as possible by an appropriate resource slice allocation rule.

Our model has been kept simple so as to expose and demonstrate our findings in accessible manner. However, it could be extended in various ways. First, we have assumed that service demand is fixed. In practice, the demand may be shaped by user behavior if users decide selfishly which SP to choose. The users' SP selection may be affected by resource availability (hence the investment vector), or by price. More elaborate infrastructure models could involve multiple different resources (*e.g.*, base stations, computing power clusters) and different service quality for users within an SP based on the resource that the user would associate to and on the number of other users using that resource. In that case, end-user competition (*a.k.a* congestion effects) would need to be modeled. Another important scenario, elaborating further the user factor, is one where SPs request resource slices by the IO by declaring their demand, and they strategize over these declarations. The sharing rule devised by the IO should then discourage dishonest reporting.

Key takeaway for CNs: The main message from this analysis is that the "case" for computing resources that are jointly contributed and shared among the contributing entities exists and a profit-sharing mechanism like Shapley can induce equilibrium states that maximize the profits of the contributing entities. This stands as an argument for CNs like guifi that actively seek for synergistic models with for-profit entities, to expand their community cloud model to include computing infrastructure providers, as they do for their network infrastructure.

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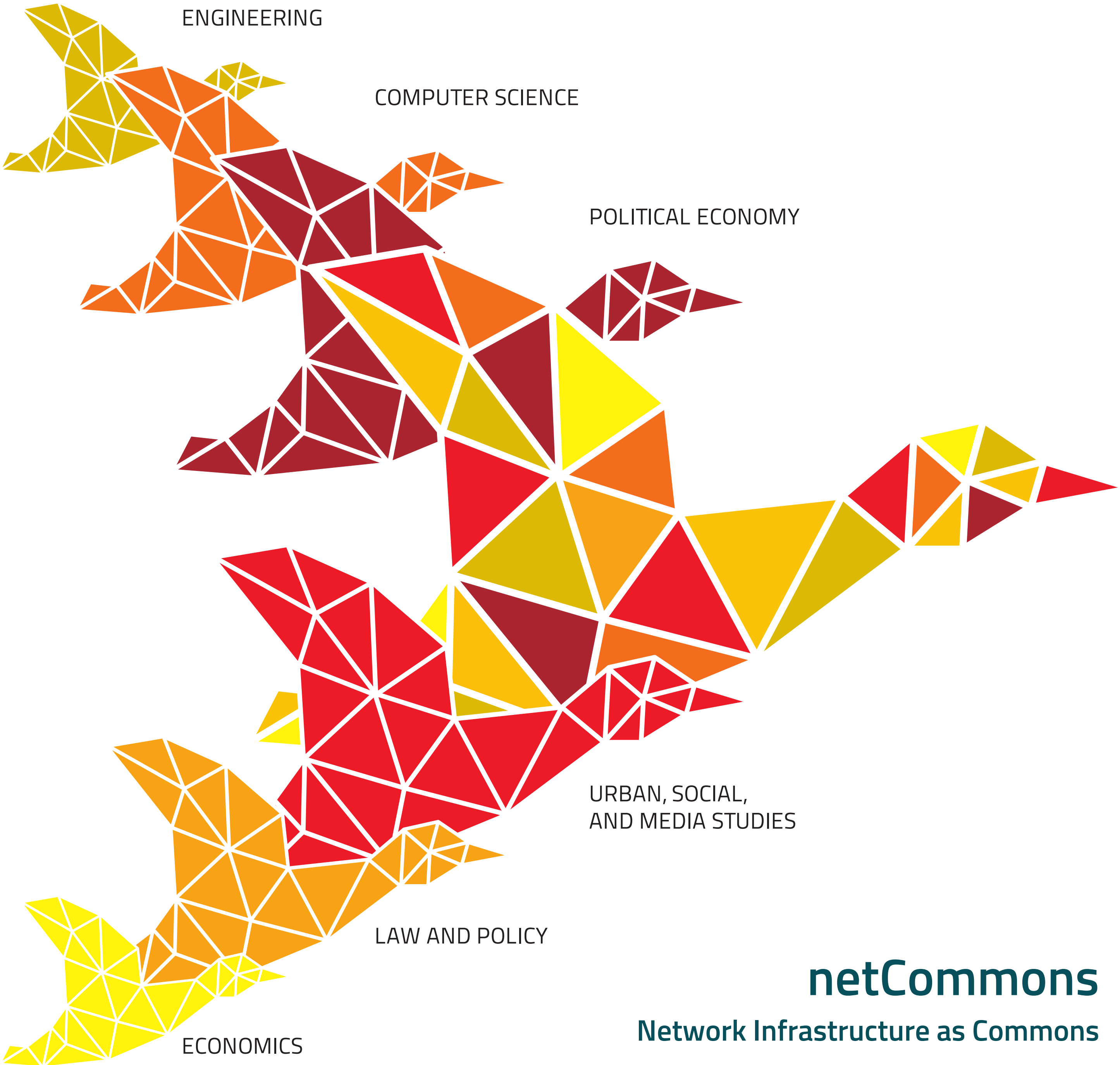
A. Appendices

As appendices to this deliverable we offer three technical documents suitable for a getting a deeper insight in the scientific work of Task 2.2. The first two are two papers that are currently under review (the first one is subject to double blind review, thus will be made available on the project web site only after the review process); the third one is the set of slides used as part of a half day tutorial, whose content was mostly formed out of the work in D2.8.

- Appendix I: M. Karaliopoulos, I. Koutsopoulos. **Collective subscriptions: towards sustainable funding of community network infrastructures**.
- Appendix II: I. Koutsopoulos, M. Karaliopoulos. **Economics of Investment and Use of Shared Network Infrastructures**.
- Appendix III: M. Karaliopoulos, I. Koutsopoulos. **Economic Sustainability in CNs and Incentives for Participation** (1.30-hr long part of a 3-hr tutorial on Wireless Community Networks and 5G: the 7-Billion-User Challenge), presented in 18th EuCNC, Ljubljana, Slovenia, June 2018.



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